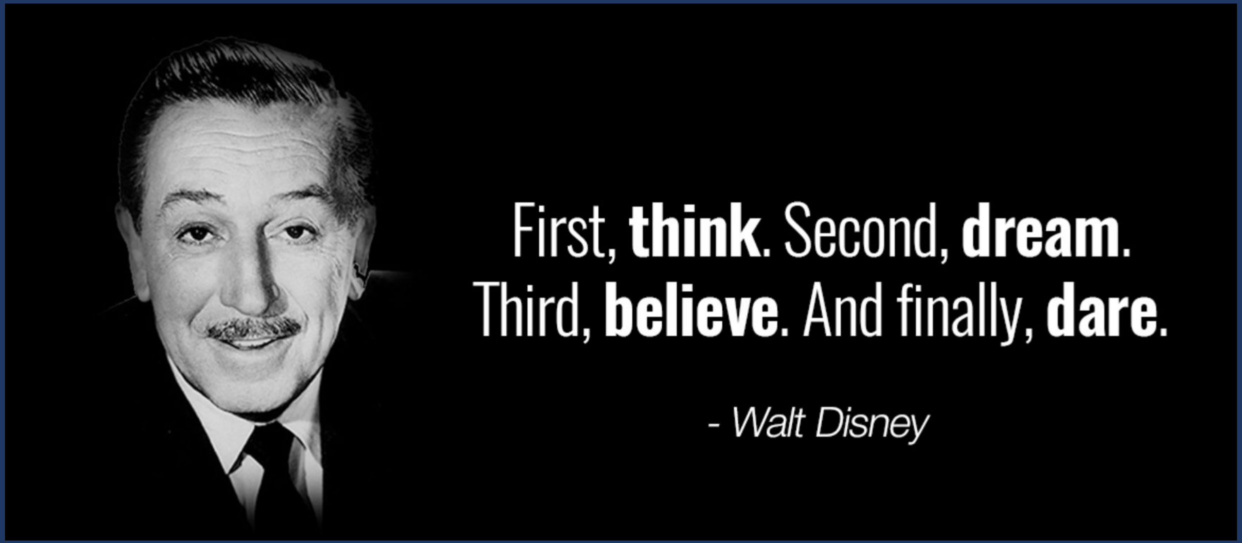
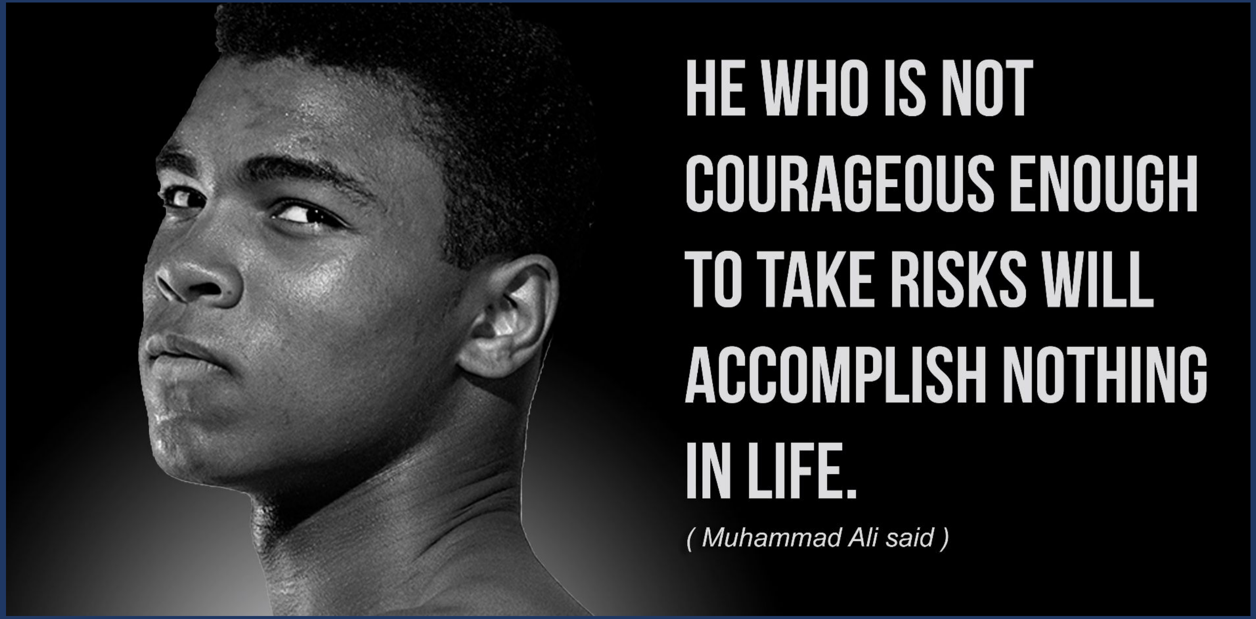
CONTROL SYSTEMS
Module 1

Markus Buchholz

HSE + demo inspirations





https://www.youtube.com/watch?v=bxbjZiKAZP4

https://www.youtube.com/watch?v=92Ye2Q8U-rY

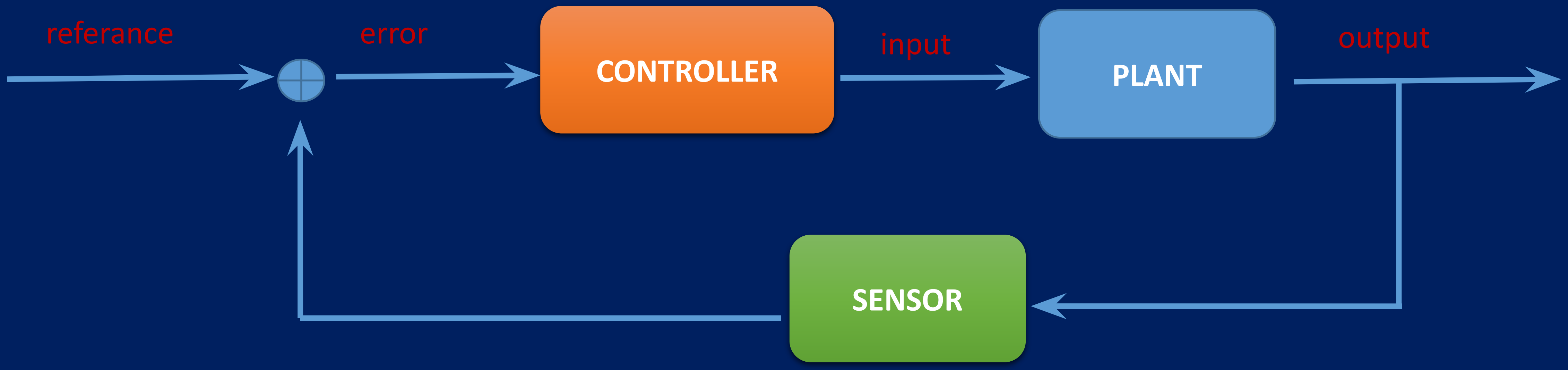
INTRODUCTION

Control System - Mechanism to change the future state of the system

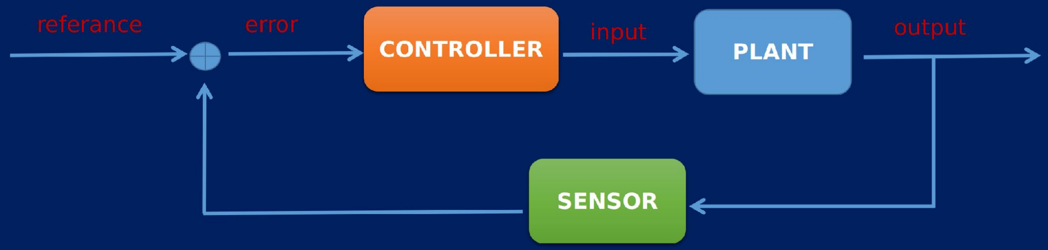
Control Therory - A strategy to select appropriate inputs

OPEN LOOP

CLOSED LOOP



CLOSED LOOP



$$E = V - y \cdot H$$

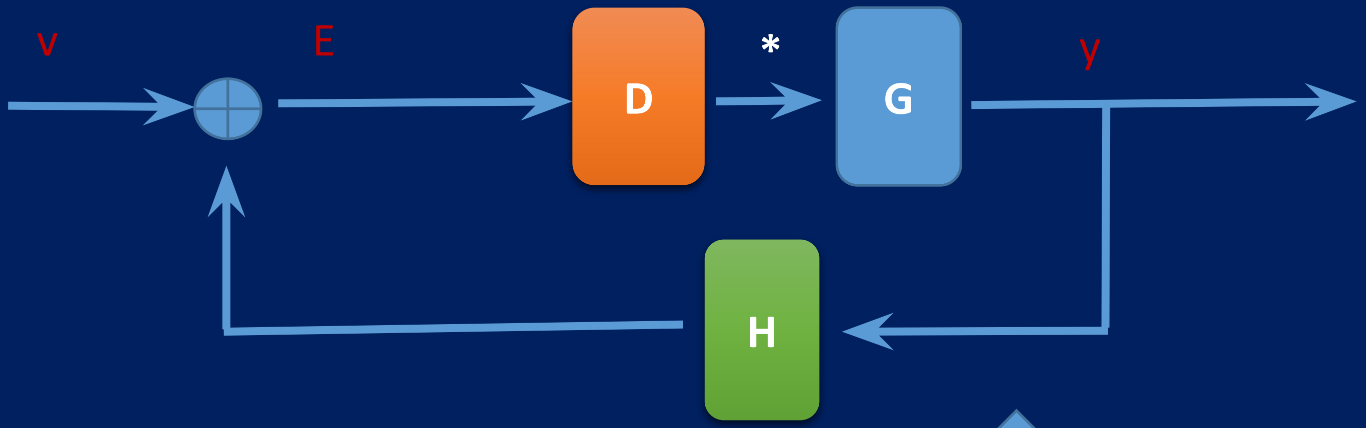
$$y = E \cdot D \cdot G \rightarrow \quad E = \frac{y}{D \cdot G}$$

$$\frac{y}{D \cdot G} = (V - y \cdot H) \quad /(D \cdot G)$$

$$D \cdot G \cdot V = y \cdot (1 + D \cdot G \cdot H)$$

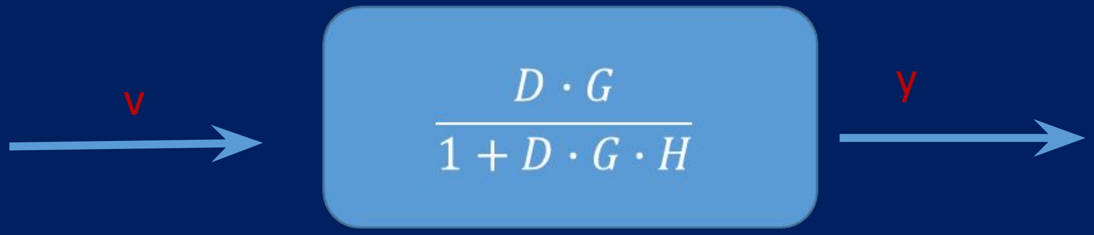
$$\boxed{y = \frac{D \cdot G}{1 + D \cdot G \cdot H} \cdot V}$$

CLOSED LOOP

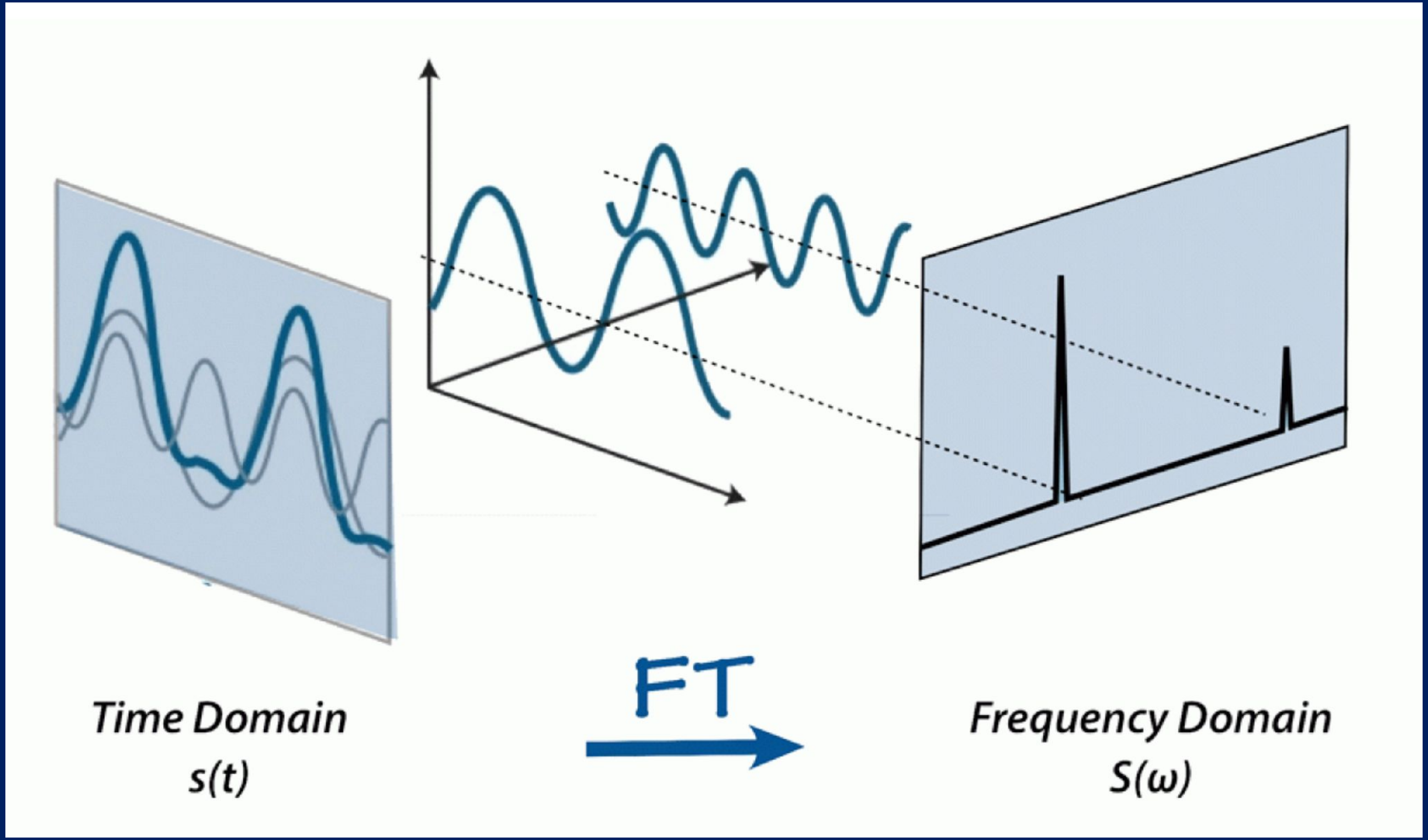


We would like to make any plant G (robot) behave like we want by adding a feedback to control system.

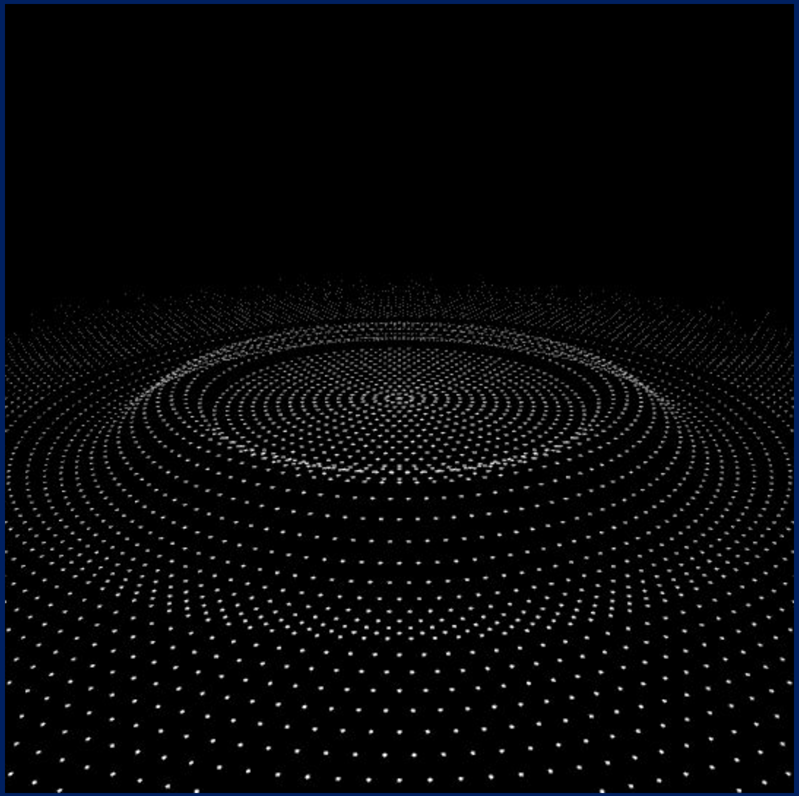
OPEN LOOP ???



TIME AND FREQUENCY DOMAIN



DIFFERENTIAL EQUATIONS



The differential equation defines a relationship between the function and its derivative (rates of change).

First order (contains the function x(t) and its derivative):

$$\frac{dy}{dx} + x = 0$$

First order system

$$where\ x = f(x)$$

Second order (contains the function y(x) and both first and second derivative):

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$$

Second order system

$$where\ y = f(x)$$

Solving the differential equation is the finding the function x(t) and y(x) that satisfy the equations.

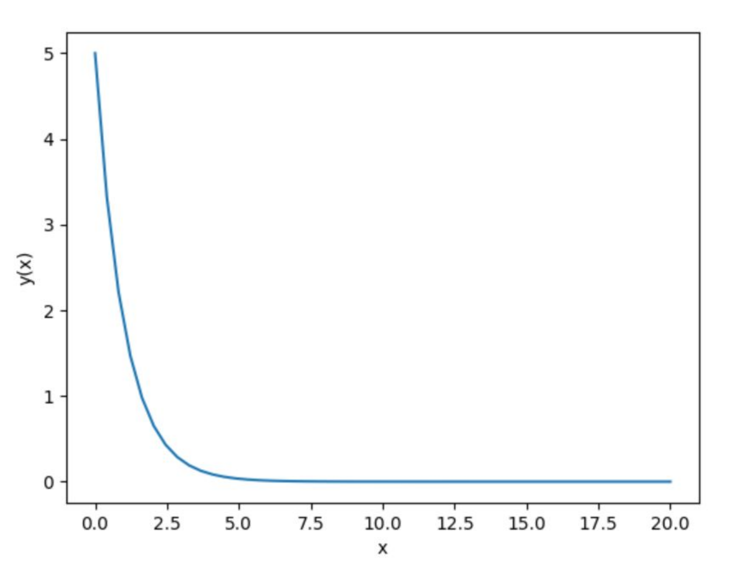
$$\frac{dy}{dx} + x = 0$$

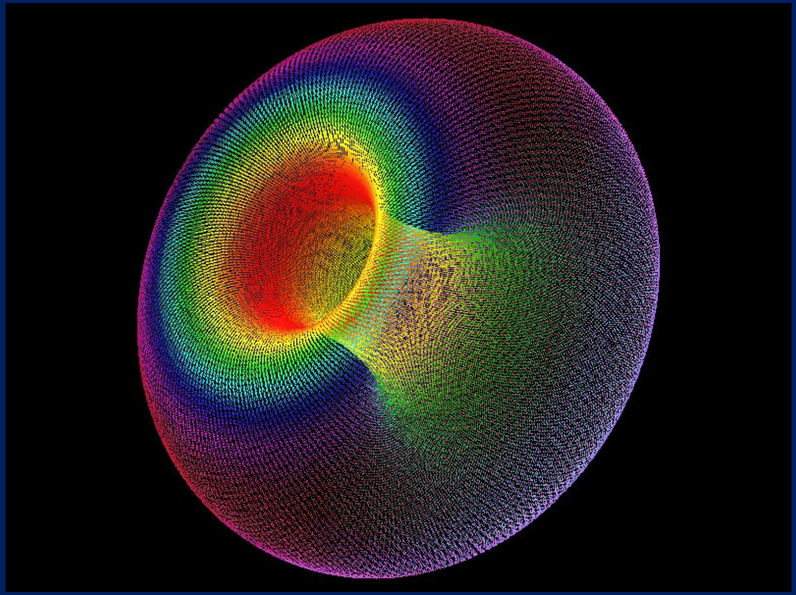
$$dy = -x \cdot dx$$

$$\int dy = -\int x \cdot dx$$

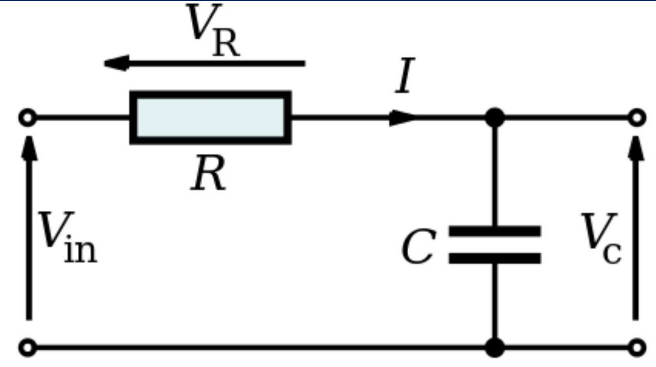
$$y + C_1 = -\frac{1}{2} \cdot x^2 + C_2$$

$$\boxed{y = -\frac{1}{2} \cdot x^2 + C_2 - C_1}$$





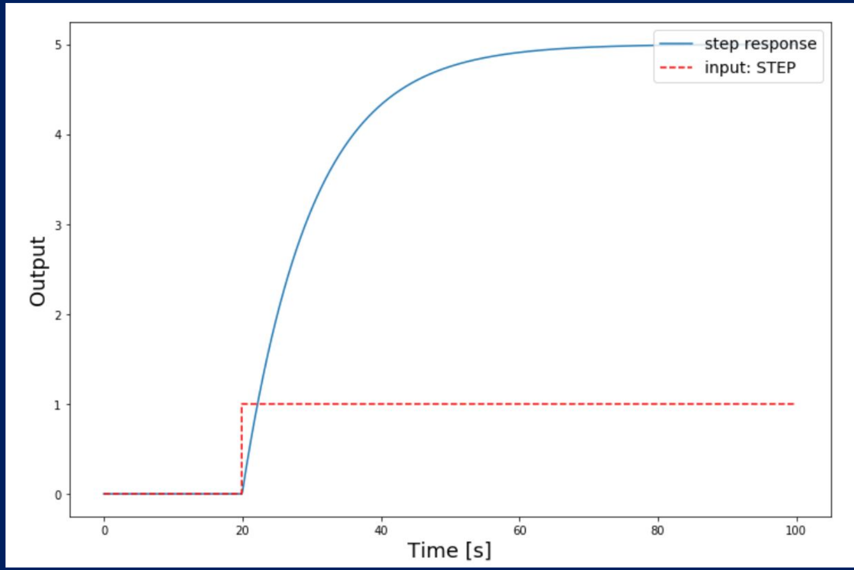
First-Order Systems (RLC)



$$\mathbf{m} \cdot \dot{\mathbf{x}} + \mathbf{k} \cdot \mathbf{x} = \mathbf{f(t)}$$

$$I(s) = \frac{C \cdot s}{1 + RCs} V(s)$$

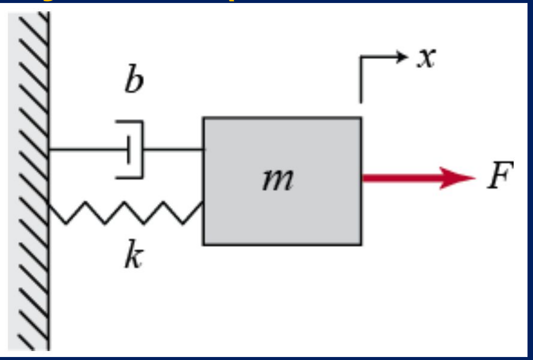
$$\frac{1.5}{10s + 1}$$



Second-Order Systems (model of robot joint)

$$\mathbf{m} \cdot \ddot{\mathbf{x}} + \mathbf{c} \cdot \dot{\mathbf{x}} + \mathbf{k} \cdot \mathbf{x} = \mathbf{f(t)}$$

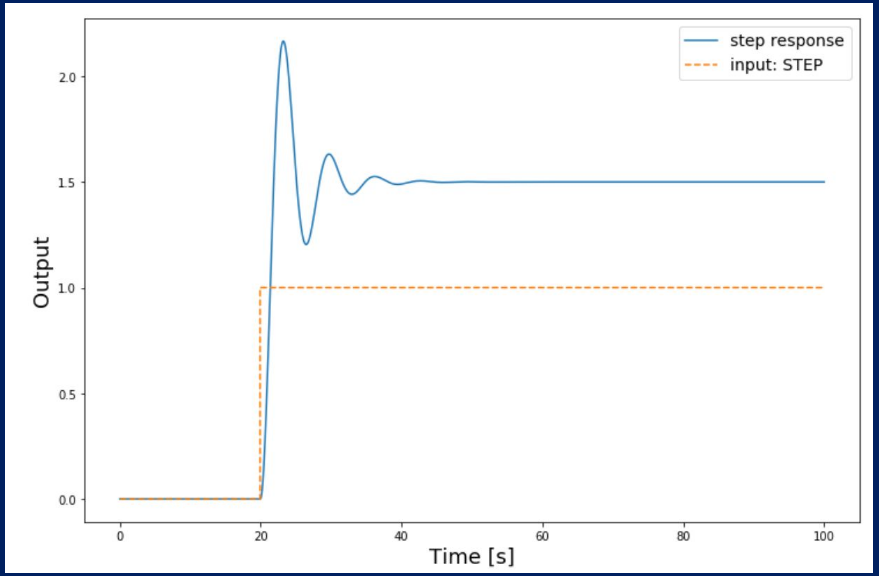
$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n{}^2 x = \frac{f(t)}{m}$$



$$\omega_n = \sqrt{\frac{k}{m}} \quad -natural\ frequency$$

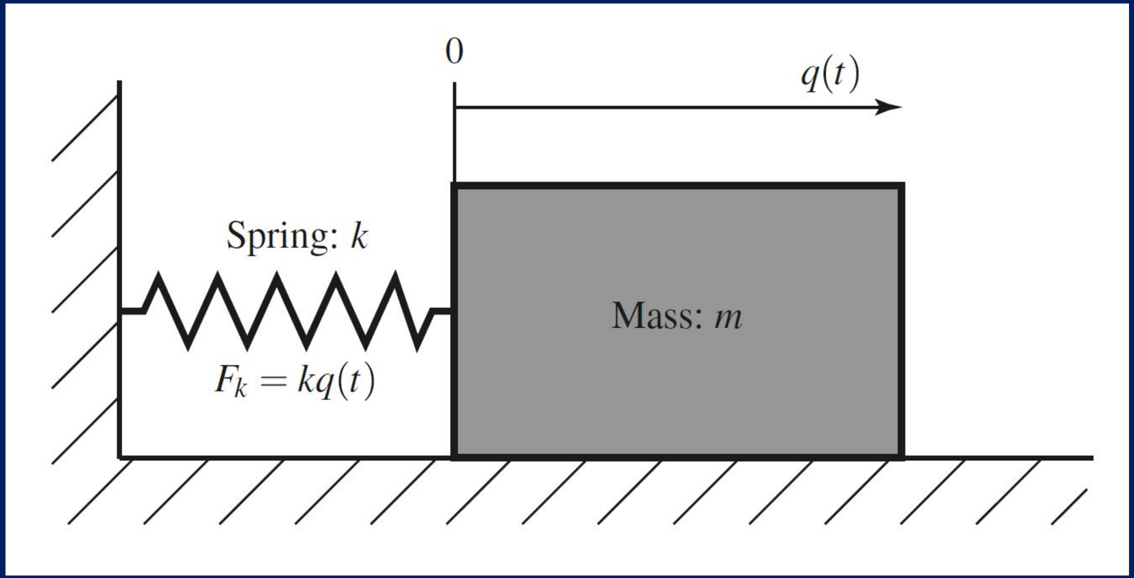
$$\zeta = \frac{c}{2\sqrt{km}} \quad -dampning$$

$$\frac{1.5}{s^2 + 0.5s + 1}$$



How to solve differential equation (second order)?

$$F = m \cdot a$$
$$F = m \cdot \ddot{q}$$
$$F = -k \cdot q$$
$$-k \cdot q = m \cdot \ddot{q}$$
$$m \cdot \ddot{q} + \mathrm{k} \cdot q = 0$$



$$\omega = \sqrt{\frac{k}{m}}$$

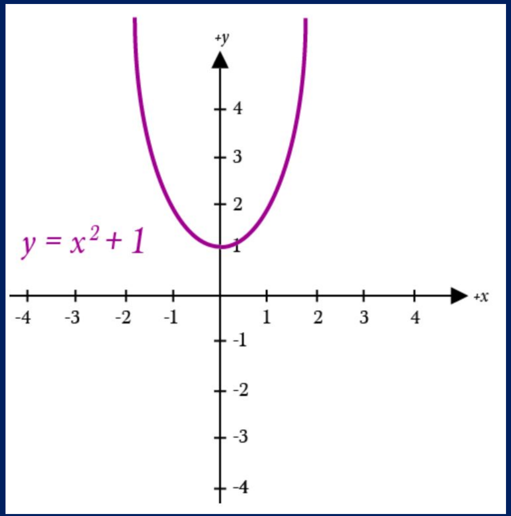
$$\ddot{q} + \omega^2 \cdot q = 0$$

The function, which solves this equation can be sine , cosine or exponential functions
(we need a function which second derivative is the original function)

$$q = \boxed{A \cdot \cos(\omega t)}$$
$$\dot{q} = -\omega A \cdot \sin(\omega t)$$
$$\ddot{q} = -\omega^2 \boxed{A \cdot \cos(\omega t)}$$
$$\ddot{q} = -\omega^2 \cdot q$$
$$\ddot{q} + \omega^2 \cdot q = 0$$

this is our
solution…

Imaginary Numbers



$$x^2 = -1 \longrightarrow x = \sqrt{-1}$$

$$x^2 = -1 \quad \Rightarrow \quad x = \sqrt{-1}$$

By Gauss:

$\boxed{i = \sqrt{-1}}$

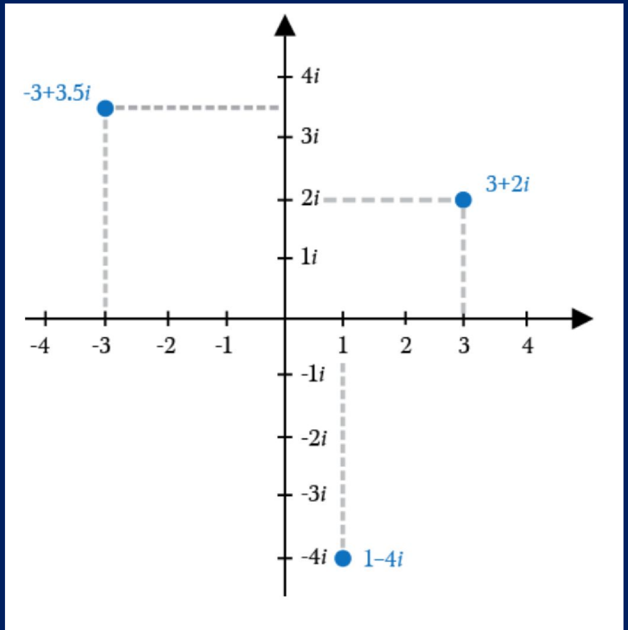
$i^2 = \sqrt{-1} \cdot \sqrt{-1} = -1$

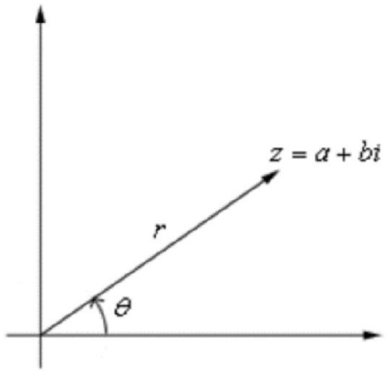
$i^3 = i^2 \cdot i = -1 \cdot i = -i$

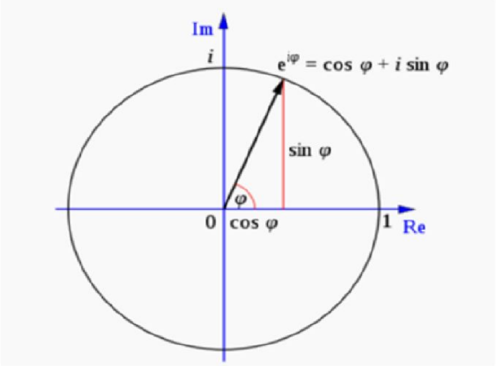
$i^4 = i^2 \cdot i^2 = (-1) \cdot (-1) = 1$

Carl Friedrich Gauss

Imaginary numbers are an extension of the reals and represents both the Amplitude and phase as they change over time.







the most remarkable formula in mathematics

$$\boxed{e^{i\pi} + 1 = 0}$$

$a = r\cos\theta \qquad b = r\sin\theta$

$$z = a + bi$$

$$z = r\left(\cos\theta + i\sin\theta\right)$$

$$\mathbf{e}^{i\,\theta} = \cos\theta + i\sin\theta$$

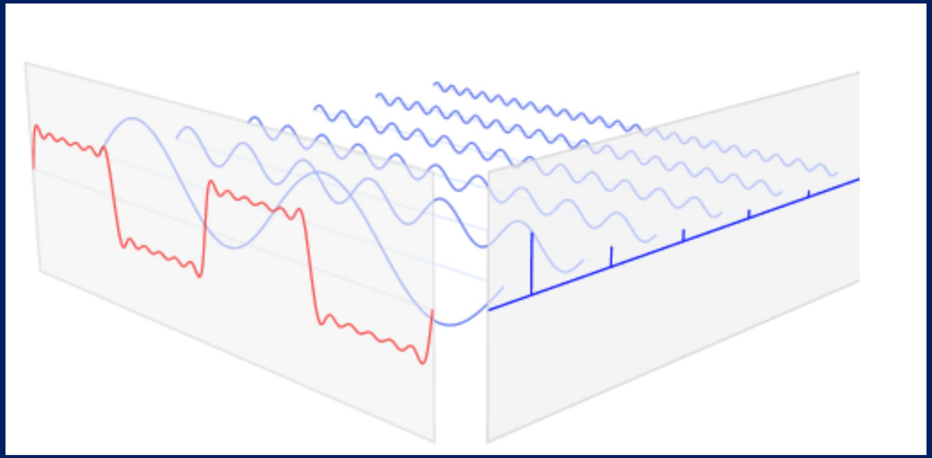
$$r = \sqrt{a^2 + b^2} \qquad \boldsymbol{r} = |z|$$

$$z = r\mathbf{e}^{i\,\theta}$$

$$z = |z|\left(\cos\theta + i\sin\theta\right) \quad \tan\theta = \frac{b}{a}$$

$$z = x + iy = |z|(\cos\varphi + i\sin\varphi) = re^{i\varphi}$$

FOURIER ANALYSIS



Jean-Baptiste Joseph Fourier

We can write any **periodic function** as an infinite sum of sines and cosines:

$$f(t) = \sum_{n=0}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

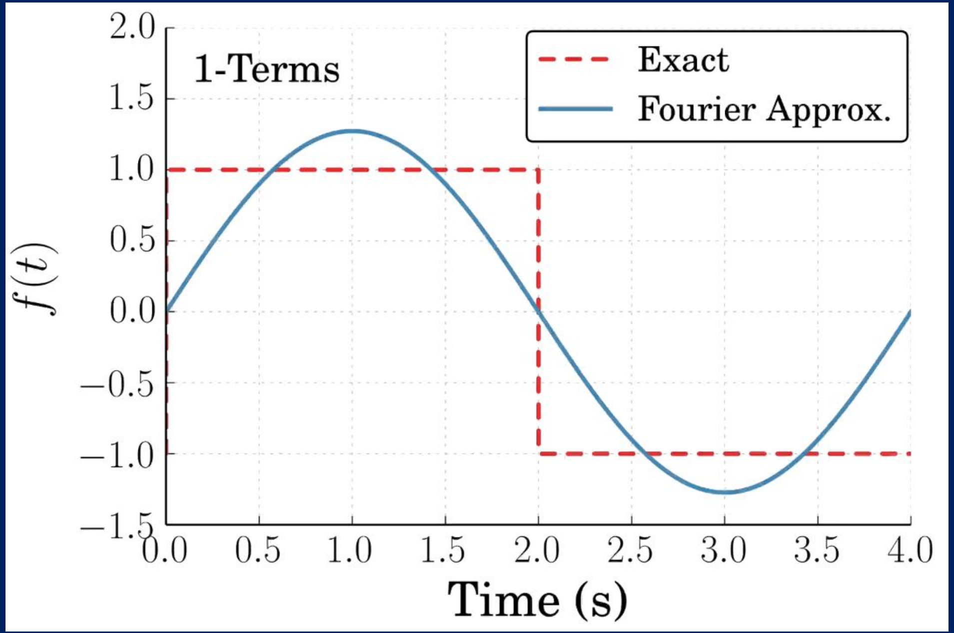
where

$$a_n = \frac{\omega_0}{\pi} \int_0^{\frac{2\pi}{\omega_0}} f(t) \cos(n\omega_0 t) dt,$$

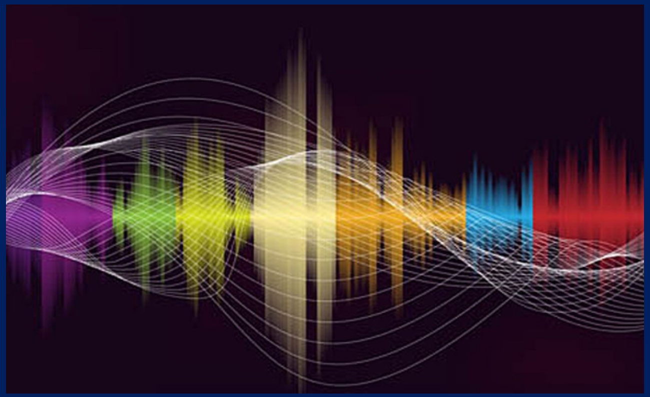
$$b_n = \frac{\omega_0}{\pi} \int_0^{\frac{2\pi}{\omega_0}} f(t) \sin(n\omega_0 t) dt,$$

and

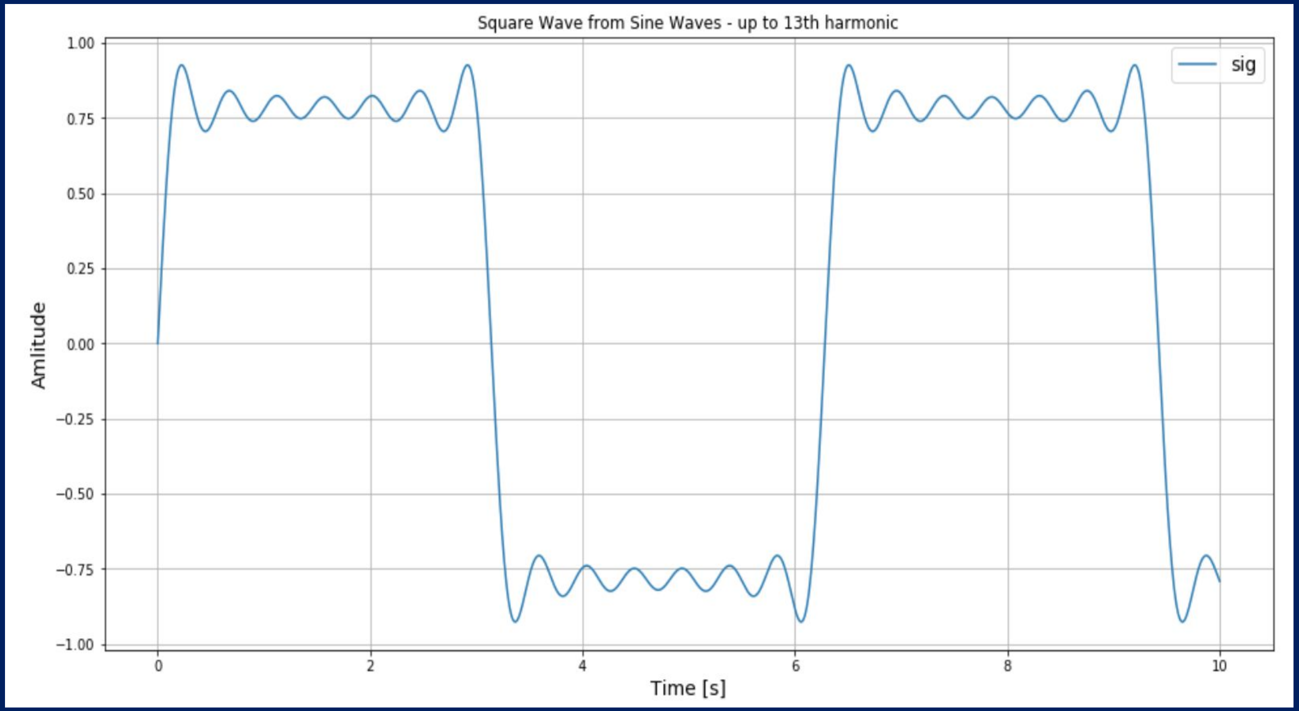
$$a_0 = \frac{\omega_0}{2\pi} \int_0^{\frac{2\pi}{\omega_0}} f(t) dt$$

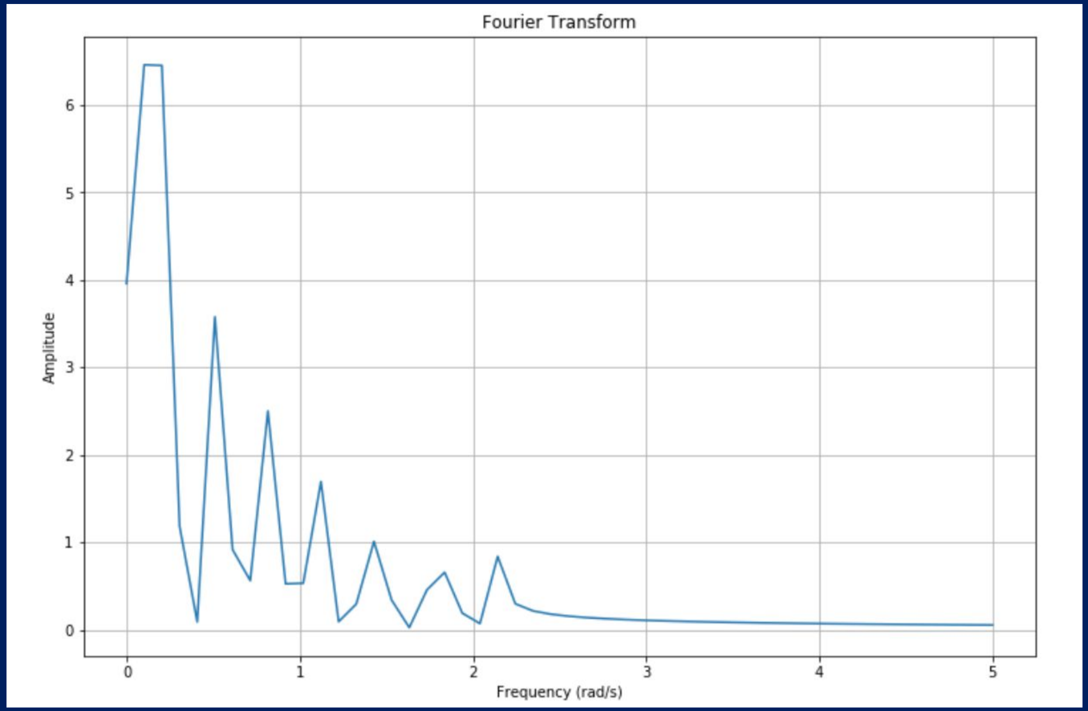


FOURIER spectrum



sig(t) = sin(t) + sin(3*t)/3 + sin(5*t)/5 + sin(7*t)/7 + sin(9*t)/9 + sin(11*t)/11 + sin(13*t)/13

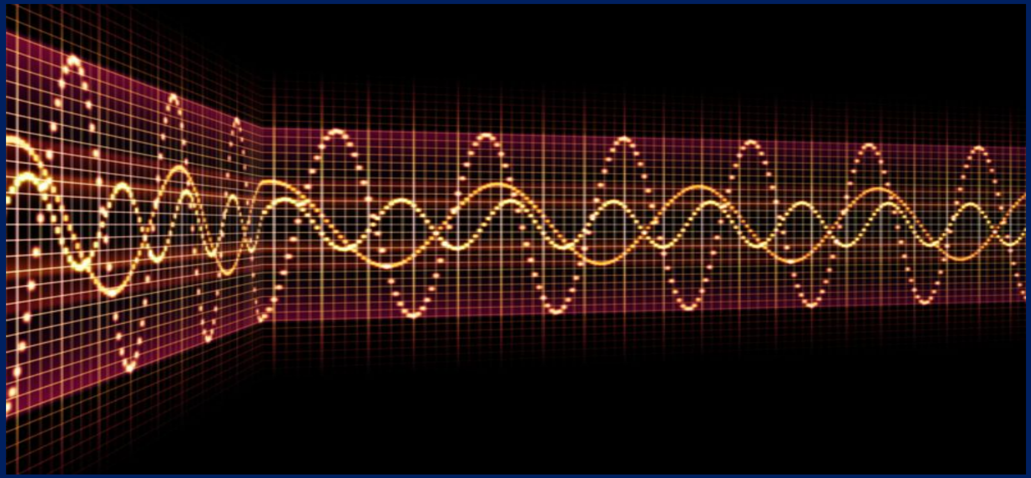




FOURIER TRANSORM

Fourier transform is a mathematical method to map **any function** in time domain into frequency domain.

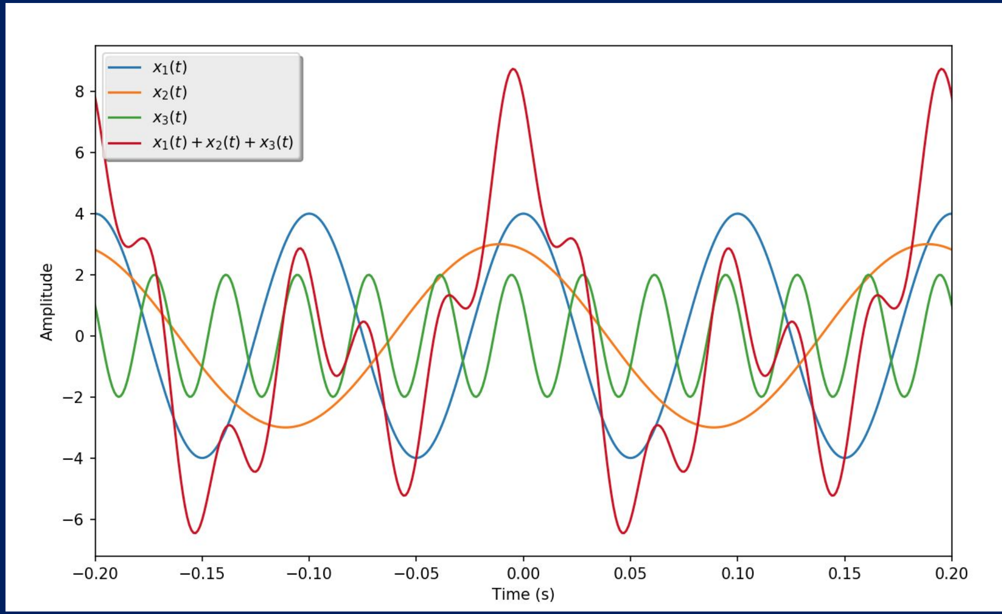
- for periodic functions, the spectrum is discrete
- for non-period functions, the spectrum is continuous

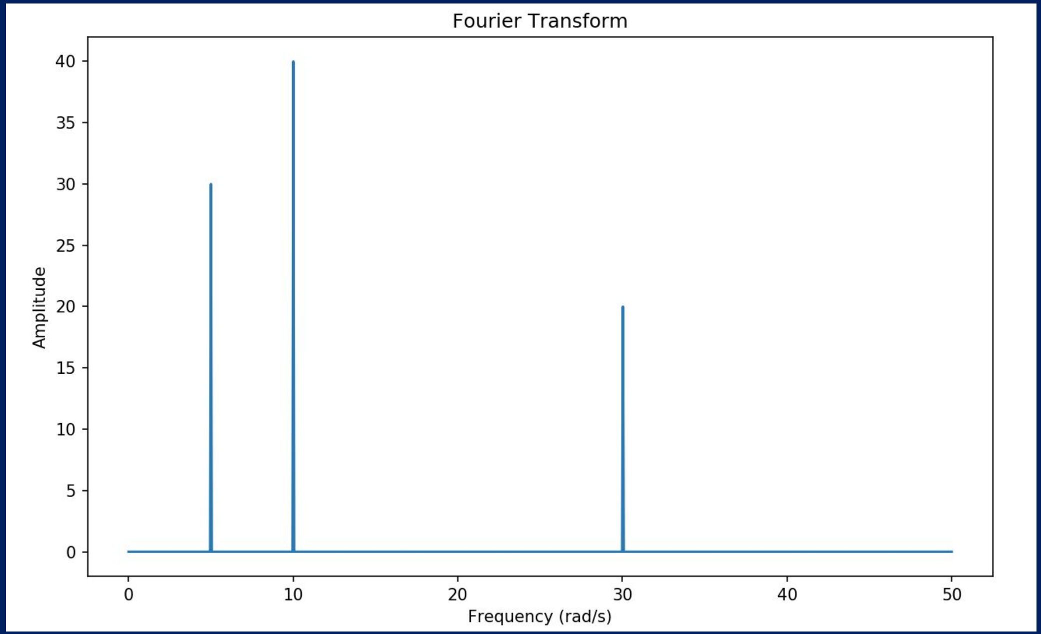


Fourier transform of a function f(t) to the frequency domain ω is defined as follows :

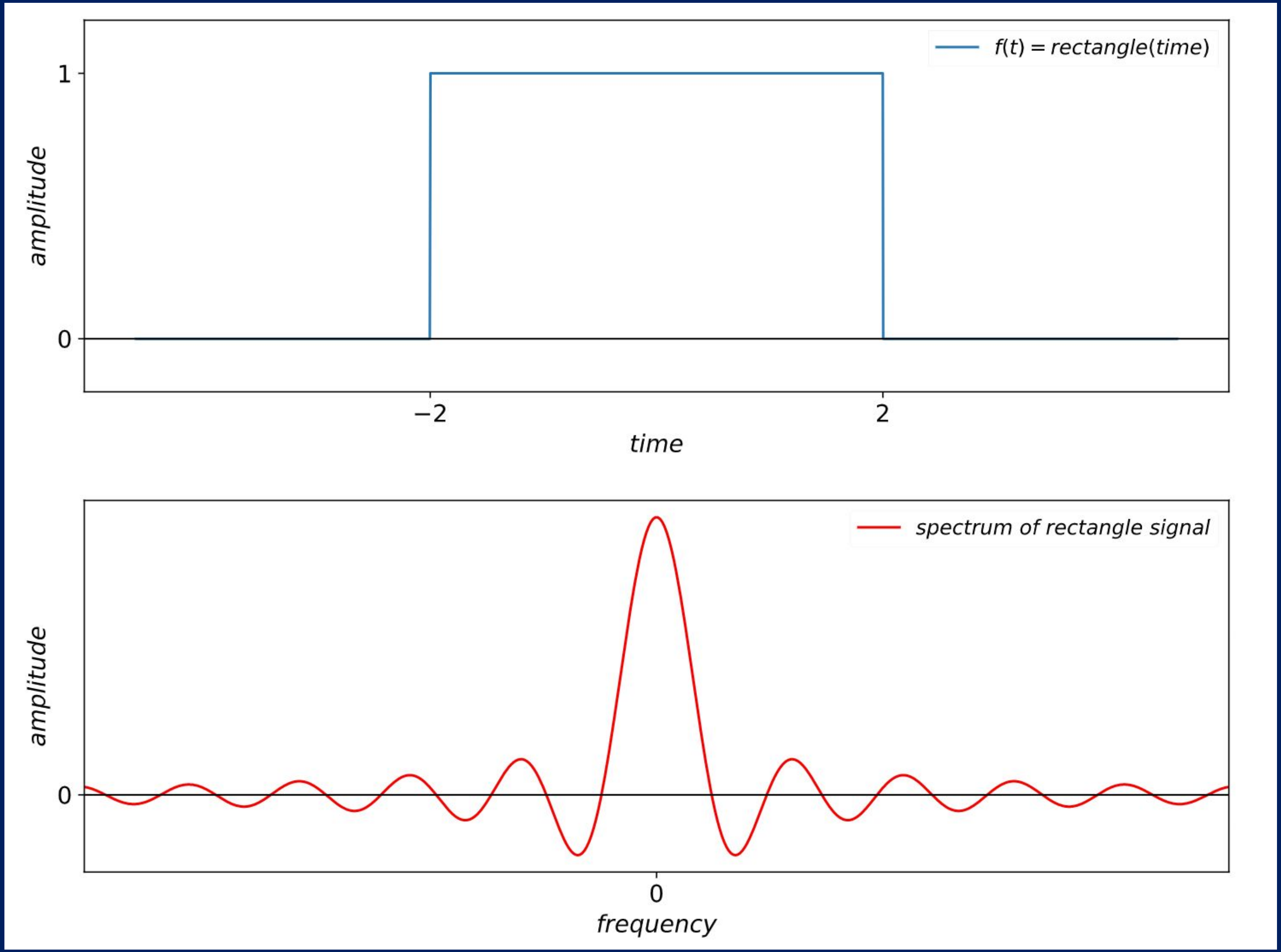
$$F(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t} f(t)dt$$

Periodic function





Non - periodic function



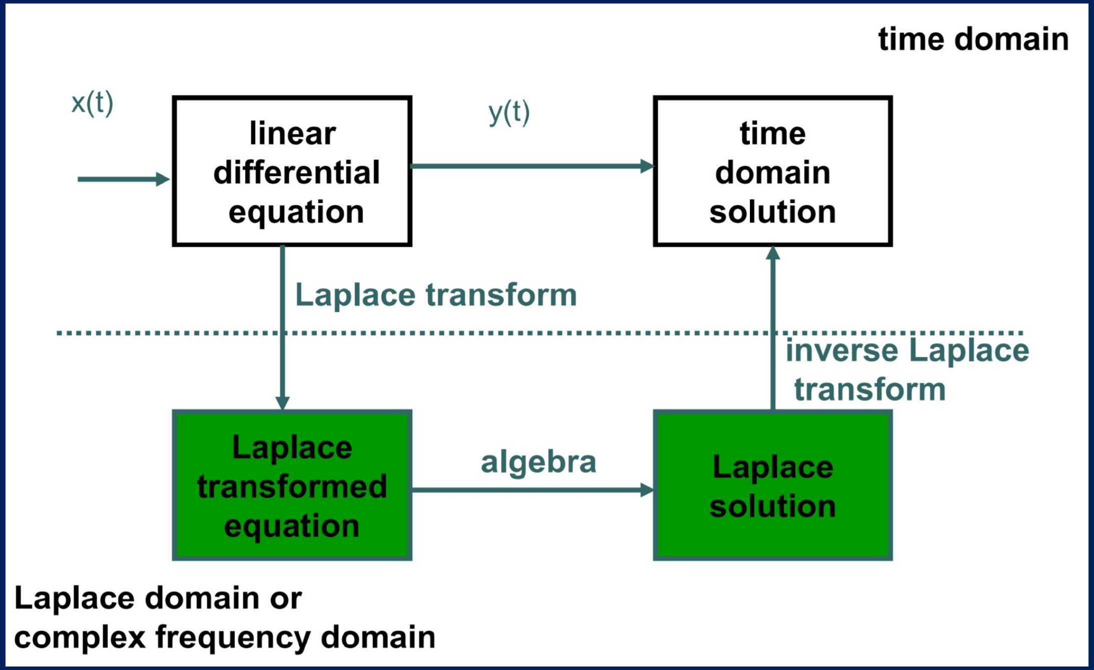
LAPLACE TRANSORM

The Laplace transform is a mathematical operation that maps a function f(t) **in the time domain to the complex frequency domain F(s)**.

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)dt$$

where s is a complex variable with
a real and imaginary part: s=σ+iωs
as opposed to t, which is a real variable.

Pierre-Simon Laplace

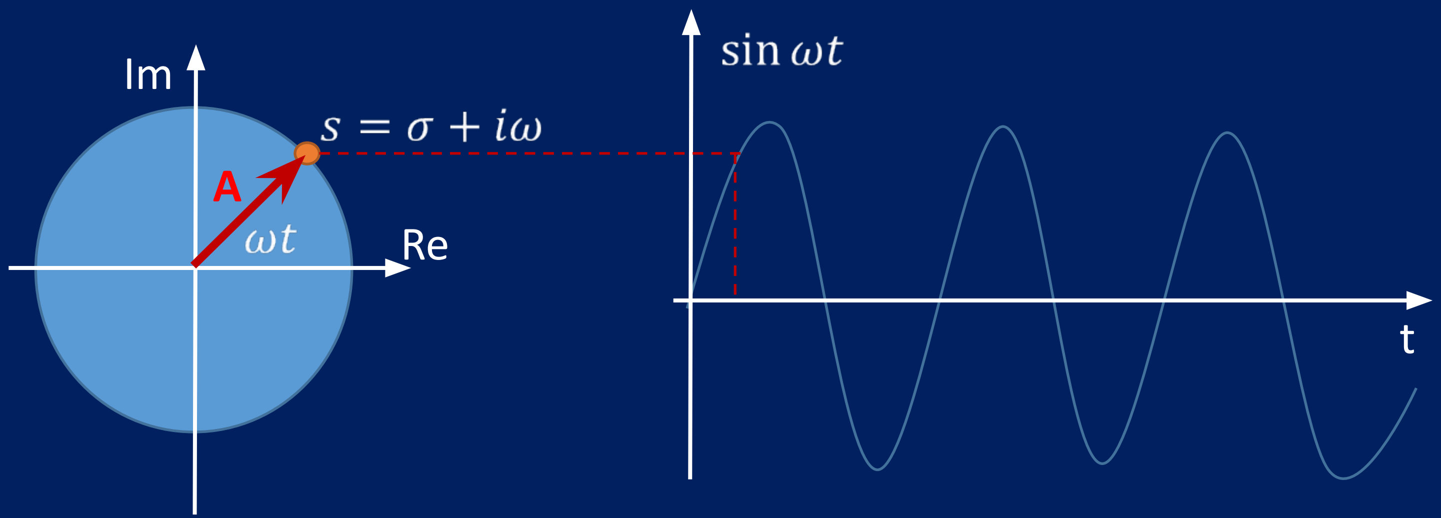


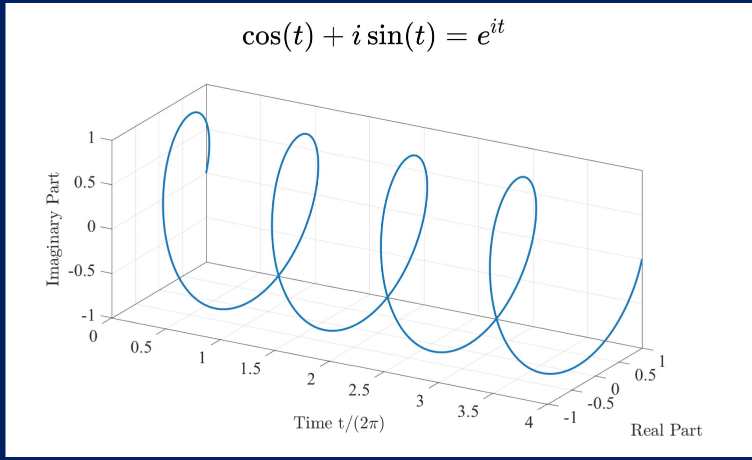
Theory (definition):

$$x(t) = A \cdot e^{st} = A \cdot e^{(\sigma+i\omega)\cdot t} = A \cdot \cos(\omega t + \sigma t) + i \cdot A \cdot \sin(\omega t + \sigma t) = A \cdot e^{\sigma t} \cdot e^{i\omega t}$$

σ – determines the rate of decay/growth – exponential responses

ω – sinusoidal responses (oscillations)





VERY IMPORTANT !!!

A linear system is described by the differential equation:

$$\boxed{\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y} = \boxed{2\frac{du}{dt} + 1}$$

Output Input

Transfer function of this system can be written:

$$H(s) = \frac{2s+1}{s^2+5s+6}$$

Input => zeros

Output =>poles

As we defined before second order system can be written:

$$\frac{d^2y}{dt^2} + 2\zeta\omega_n\frac{dy}{dt} + \omega_n{}^2x = 0$$

$$\omega_n = \sqrt{\frac{k}{m}} \ -natural\ frequency$$

$$\zeta = \frac{c}{2\sqrt{km}} \ -dampning$$

and solutions,

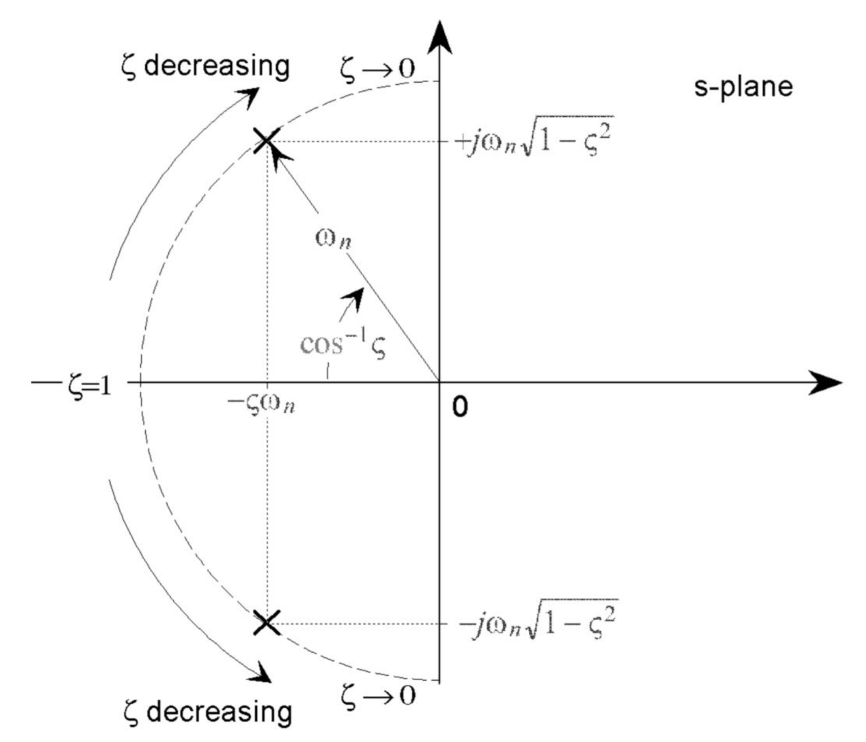
$$p_1, p_2 = -\zeta\omega_n \pm j\omega_n\sqrt{\zeta^2 - 1}$$

$\zeta < 1$ – underdamped system

$\zeta \geq 1$ – overdamped system

$\zeta = 0$ – undamped system

$\zeta = 1$ – criticaly damped system



solutions: $p_1, p_2 = -\zeta\omega_n \pm j\omega_n\sqrt{\zeta^2 - 1}$ can be also written as (we know from previous slide)

$$p_1, p_2 = \alpha \pm j\beta$$

$$x(t) = C_1 e^{p_1 t} + C_2 e^{p_2 t} = C_1 e^{\alpha t} \cdot \cos(\beta t) + C_2 e^{\alpha t} \cdot \sin(\beta t)$$

$$C_1 = r\cos(\theta), \quad C_2 = r\sin(\theta)$$

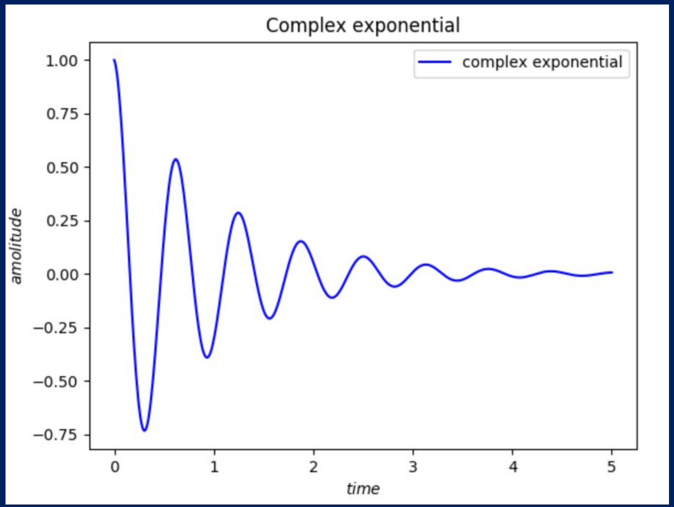
$$r = \sqrt{{C_1}^2 + {C_2}^2}$$

$$\theta = \mathrm{Atan2}(\frac{C_2}{C_1})$$

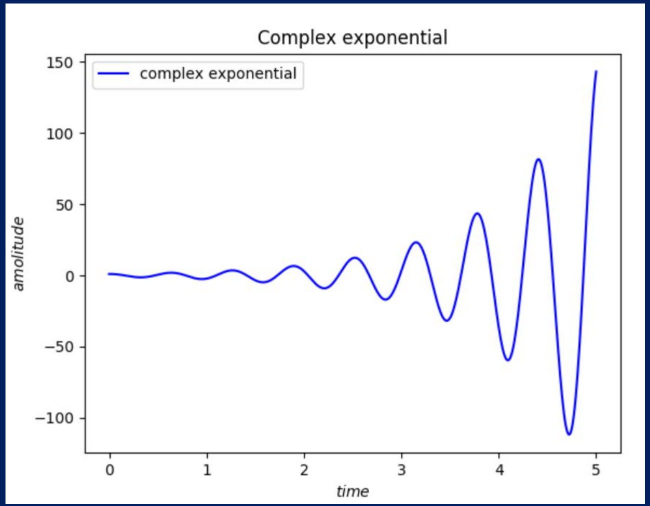
1. **We are looking for systems which are STABLE (checking system stability).**
2. **System after applying input signal regaining to previous state.**
3. **We are interesting mainly LEFT side of REAL axis of f(s).**

$$\boldsymbol{x(t) = re^{\alpha t}\cos(\beta t - \theta)} \qquad \boldsymbol{e^{\alpha t} = e^{-\zeta\omega_n t}}$$

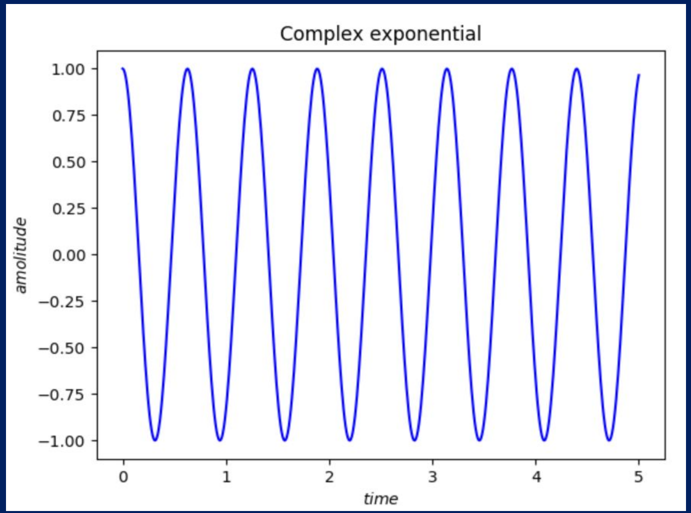
$\alpha < 0$



$\alpha > 0$



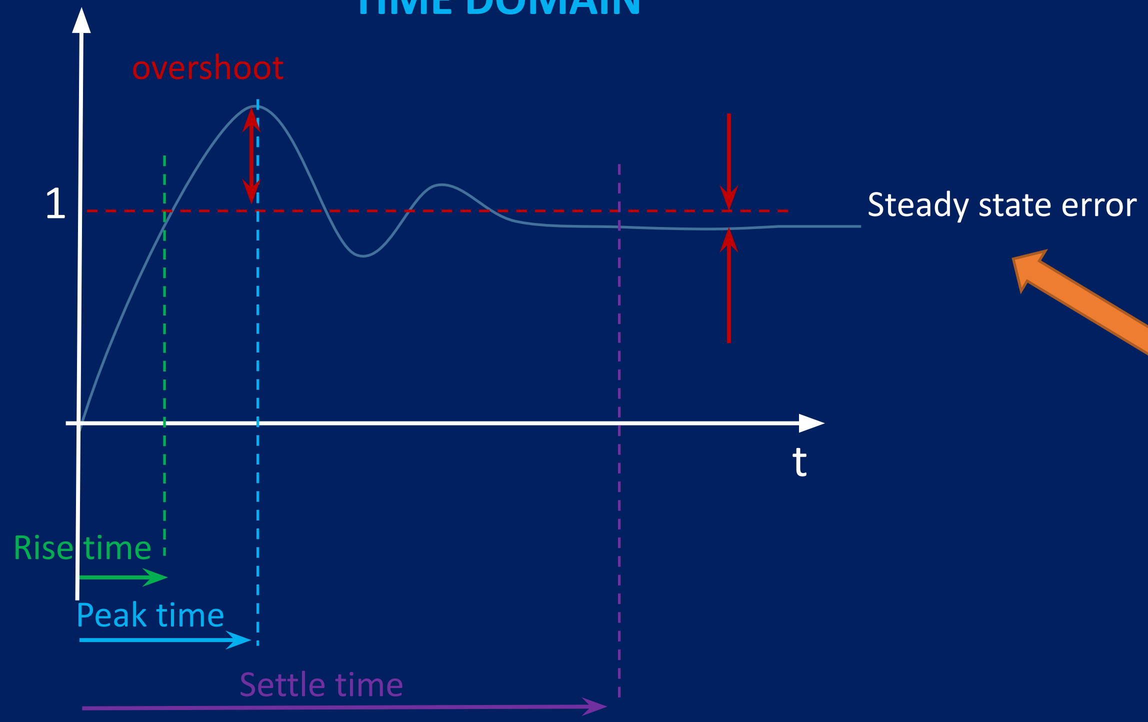
$\alpha = 0$



VERY IMPORTANT !!!

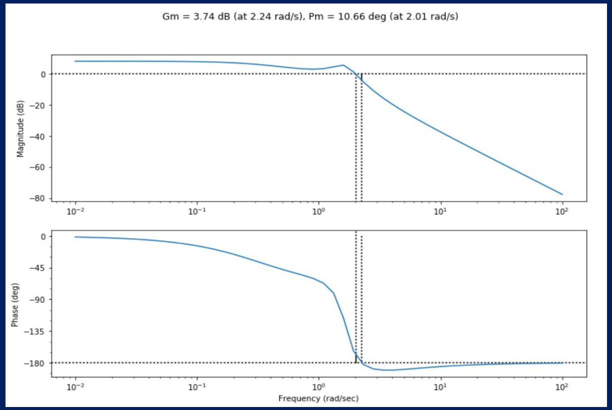
DOMAINS WE CONSIDER IN CONTROLS SYSTEM (How we design, what we are looing for?)

TIME DOMAIN



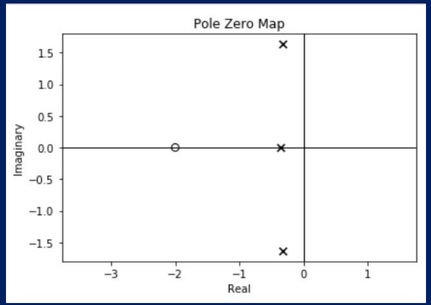
FREQUENCY DOMAIN

- Bandwidth
- Gain margin
- Phase margin



S - DOMAIN

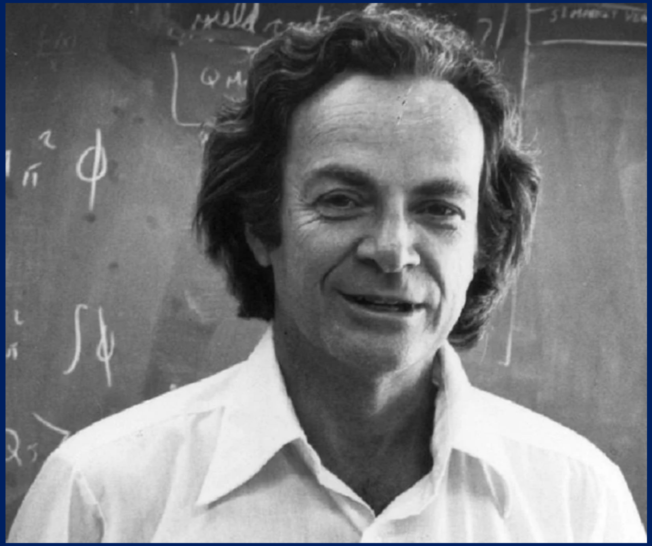
Pole and zero locations



Why do we apply Laplace transform?

Linear and Time Invariance – LTI systems – are systems which respond in certain way when subjected to arbitrary input.

" Linear systems are important because we can solve them " – R. Feynmam



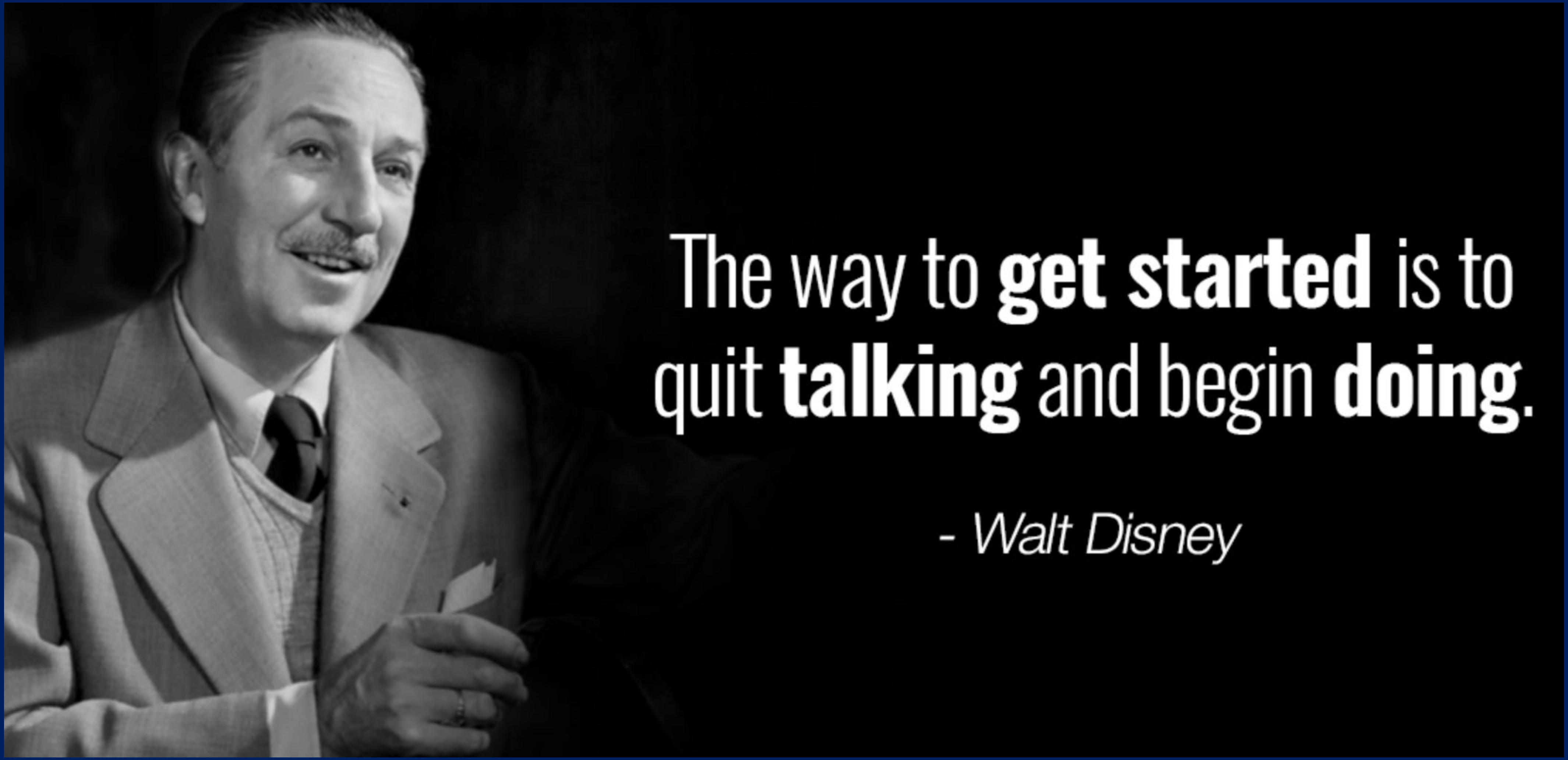
Richard Feynman

Linear Time Invariant, or LTI processes have three properties.

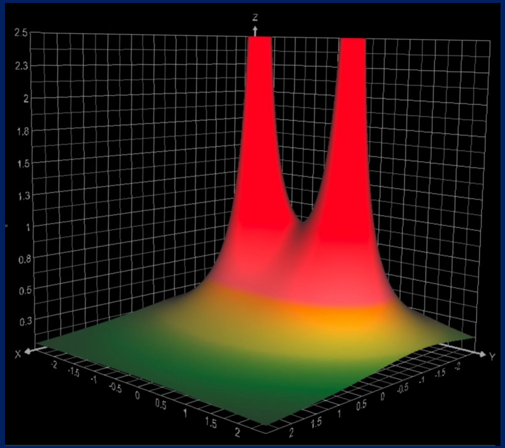
1. **Homogeneity:** An input u(t), to an LTI system scaled by α will have an output y(t), scaled by α.

2. **Additivity (Superposition):**
   a. If an input u1(t) to an LTI system produces an output y1(t) and if an input u2(t) to an LTI system produces an output y2(t)
   b. Then the final output from the summed input of u1(t)+u2(t) is the summed output of y1(t)+y2(t).

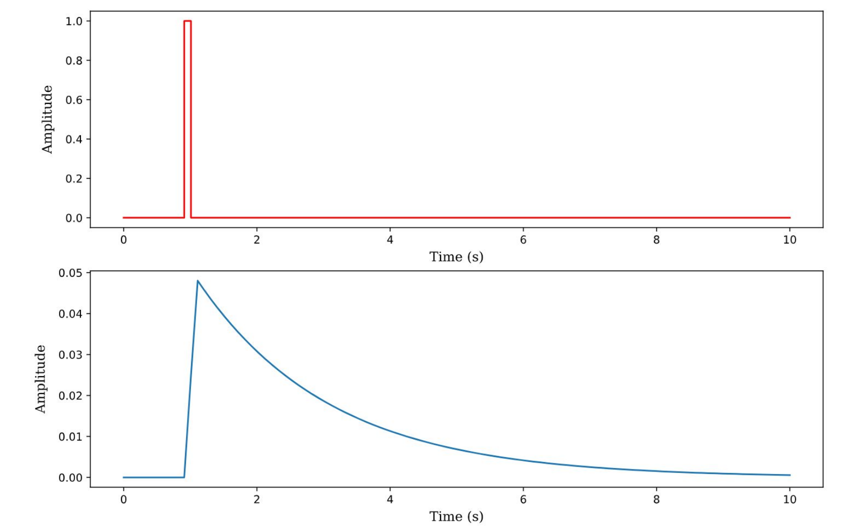
3. **Time Invariance:** If an input u(t) to an LTI system produces an output y(t) then an input translated in time (e.g. the same input that happens after a time units) u(t−a), will produce an output y(t−a) that translated in time.


The way to get started is to
quit talking and begin doing.
- Walt Disney

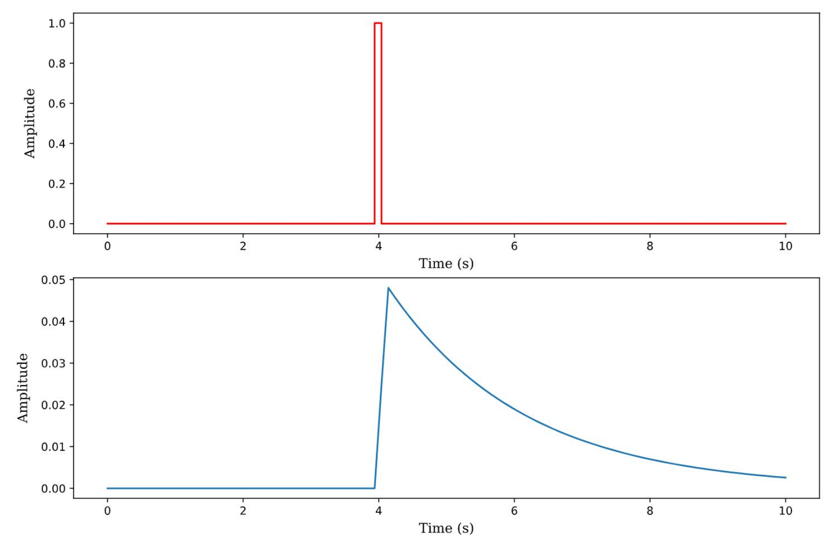
Example for Additivity (Superposition)



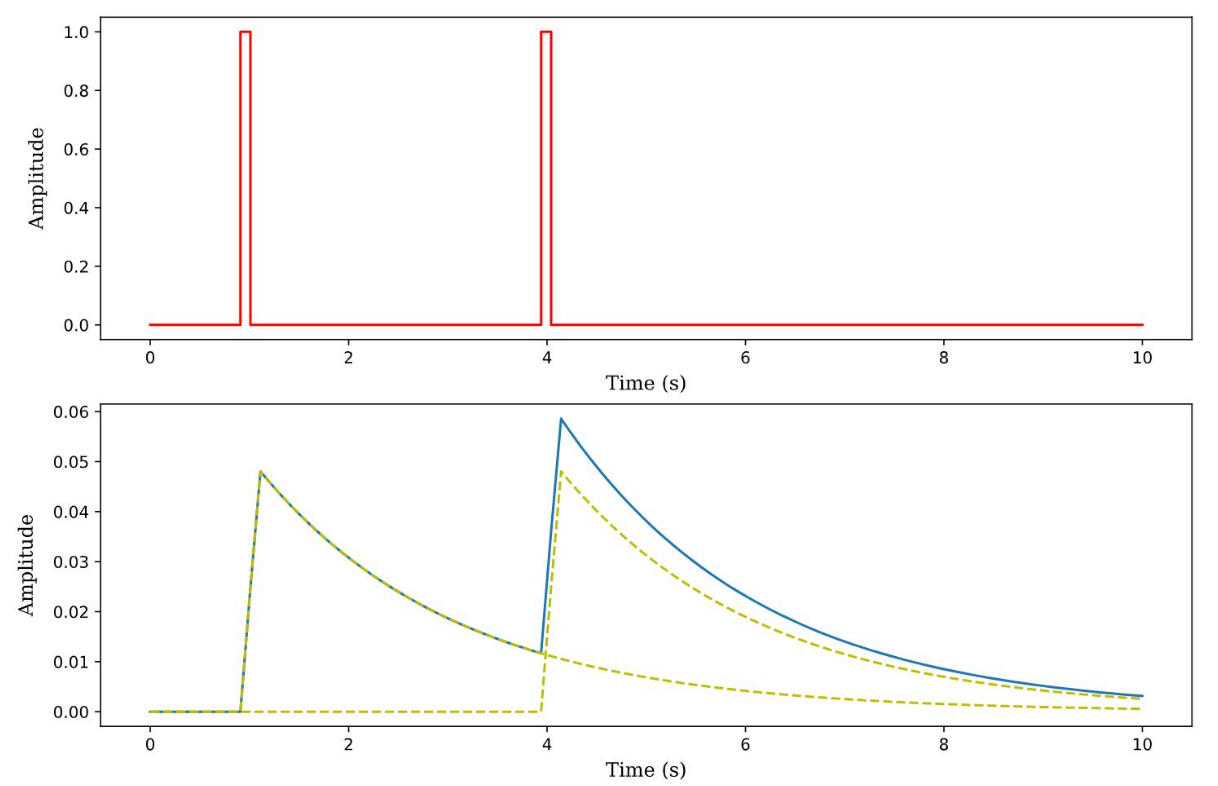
y1(t)



y2(t)



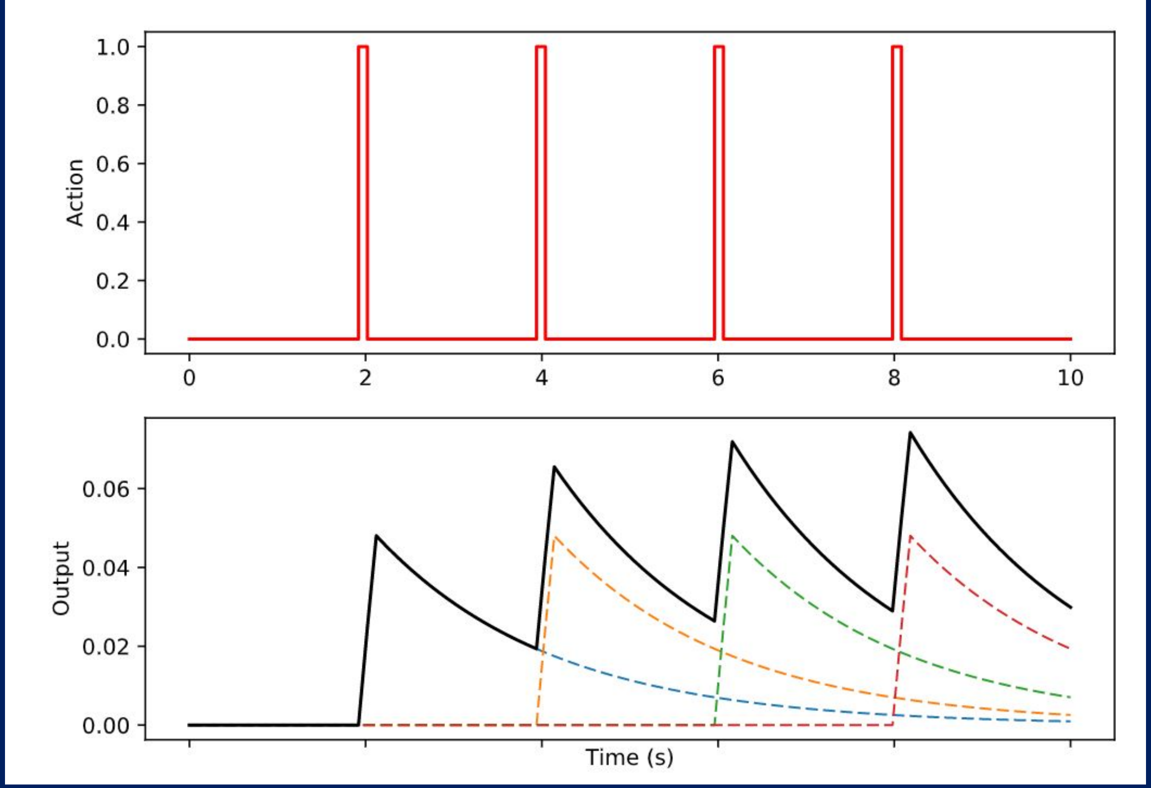
y1(t)+y2(t)



If the number of such impulses increase (to large number/infinity) so the output responses (from the system) will be difficult to add in time domain – **in time domain we use convolution**.

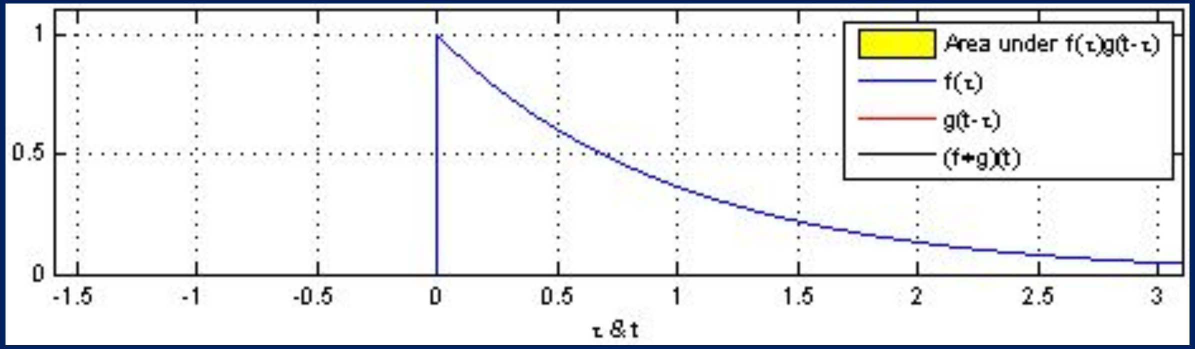
In s domain (Laplace) we use multiplication!!!
Math to compute differential equations (in time domain) is simplified (in s–domain we use Algebra).

=> **this is answer why we apply Laplace transform**.



Signals convolution

$$(f * g)(t) \triangleq \int_{-\infty}^{\infty} f(\tau)g(t-\tau)\,d\tau$$



How we use Laplace transform?

$$\mathcal{L}(f;s) = \int_0^\infty \mathrm{e}^{-st} f(t)\,dt$$

$$f(t) = b$$

$$\mathcal{L}(b;s) = \int_0^\infty b\mathrm{e}^{-st}\,dt = \left.\frac{b\mathrm{e}^{-st}}{-s}\right|_0^\infty = \begin{cases} \frac{b}{s} & \text{if } \mathrm{Re}(s) > 0 \\ \text{divergent} & \text{otherwise} \end{cases}$$

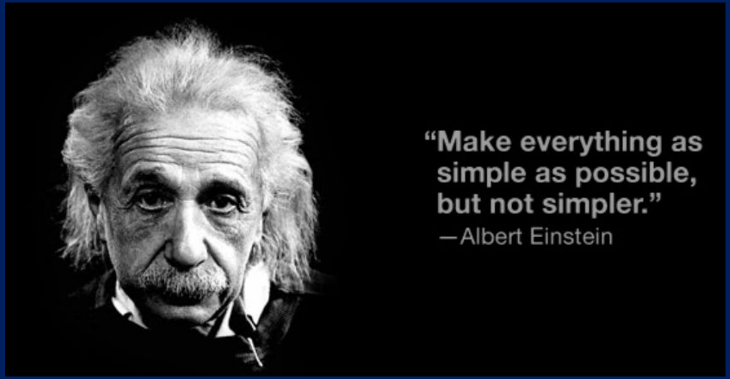
$$f(t) = \mathrm{e}^{at}$$

$$\mathcal{L}(\mathrm{e}^{at};s) = \int_0^\infty \mathrm{e}^{at}\mathrm{e}^{-st}\,dt = \int_0^\infty \mathrm{e}^{(a-s)t}\,dt = \left.\frac{\mathrm{e}^{(a-s)t}}{a-s}\right|_0^\infty = \begin{cases} \frac{1}{s-a} & \text{if } \mathrm{Re}(s) > \mathrm{Re}(a) \\ \text{divergent} & \text{otherwise} \end{cases}$$

Laplace transforms table

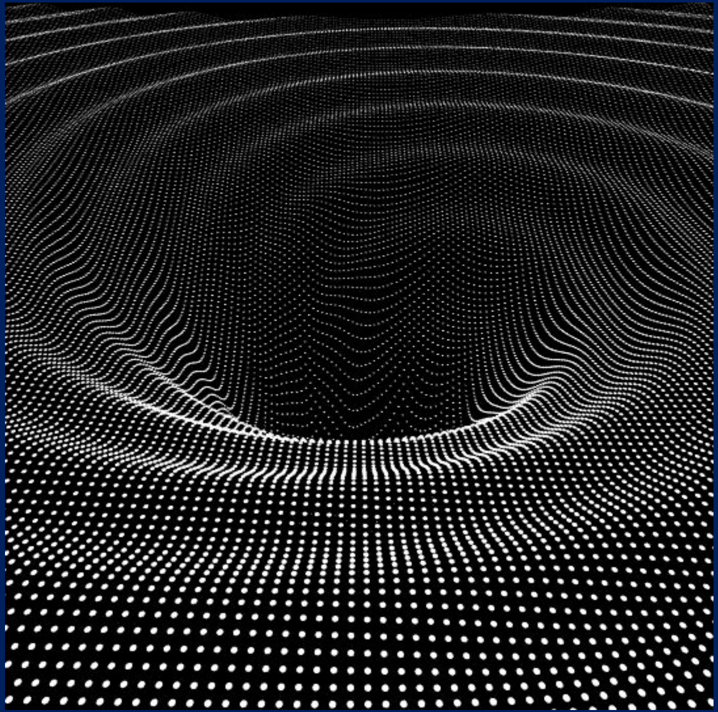
$f(t)$	$\mathcal{L}[f(t)]$
1	$\frac{1}{s}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$t\sin at$	$\frac{2as}{(s^2+a^2)^2}$
$t\cos at$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$
$\frac{t^n}{n!}, n \in \mathbb{N}$	$\frac{1}{s^{n+1}}$
$e^{at}\cdot\frac{t^n}{n!}, n \in \mathbb{N}$	$\frac{1}{(s-a)^{n+1}}$

We use Laplace transform to solve differential equations.



Laplace derivatives of function f(t):

$$\mathcal{L}(f'(t);s) = sF(s) - f(0)$$
$$\mathcal{L}(f'';s) = s^2F(s) - sf(0) - f'(0)$$
$$\mathcal{L}(f''';s) = s^3F(s) - s^2f(0) - sf'(0) - f''(0)$$

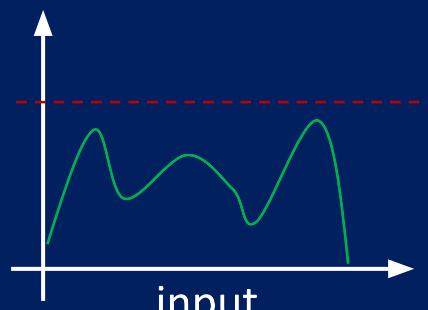


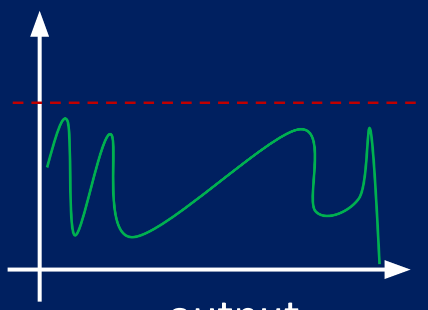
Solve $y'' - y = e^{2t}$, $y(0) = 1$, $y'(0) = 1$

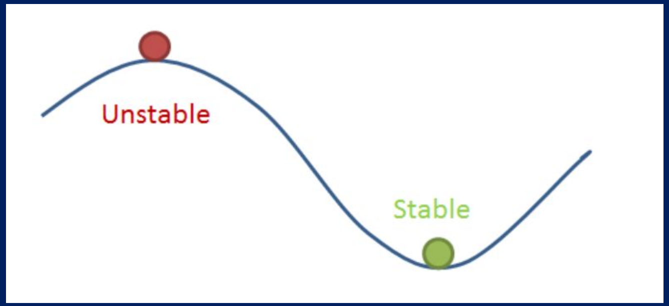
$$(s^2Y - sy(0) - y'(0)) - Y = \frac{1}{s-2}$$
$$(s^2 - 1)Y = \frac{1}{s-2} + s + 1$$
$$Y = \frac{1}{(s-2)(s^2-1)} + \frac{s+1}{s^2-1} = \frac{1}{(s-2)(s^2-1)} + \frac{1}{s-1}$$

We do we apply Control System?

We need BIBO stable systems (bounded-input, bounded-output).







MODELING

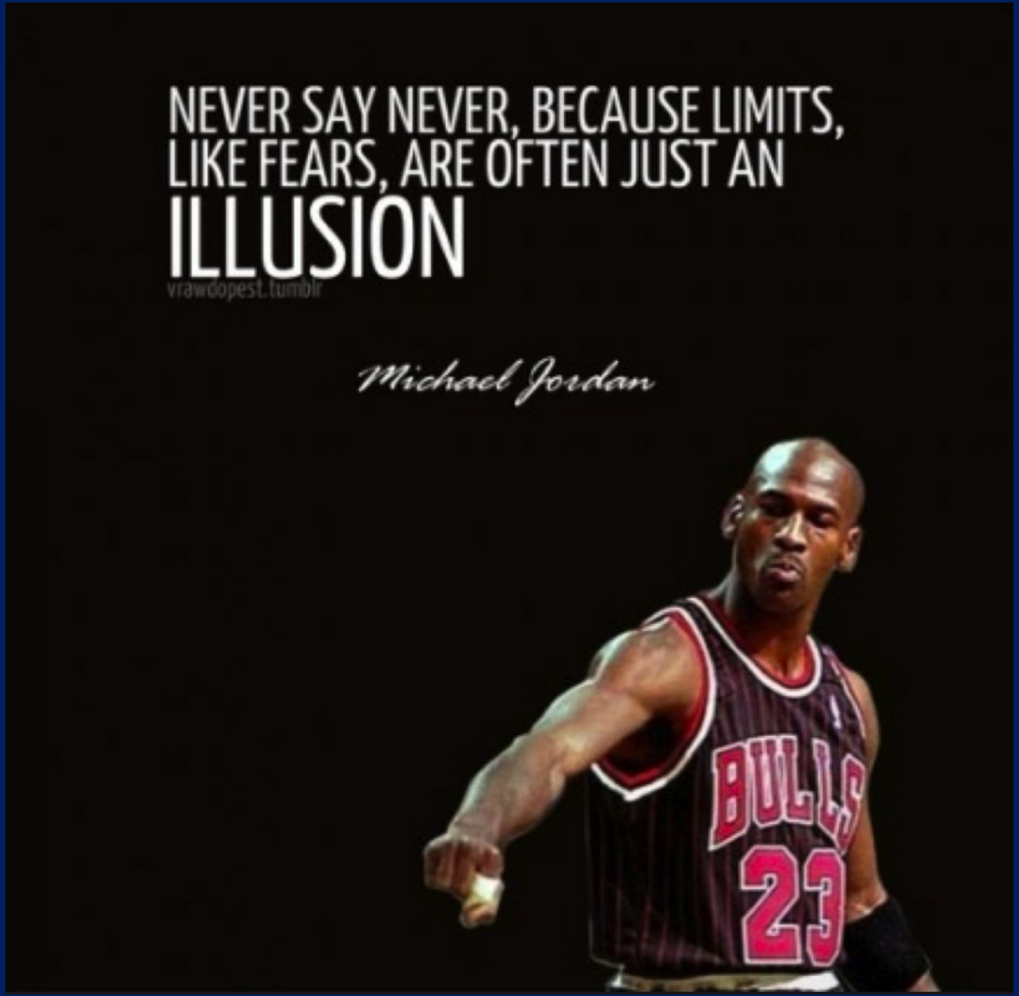
Model of System
Differential Equations
Laplace Transform
State Space
Block Diagrams
Discrete Time Differential Equations

ANALYSIS

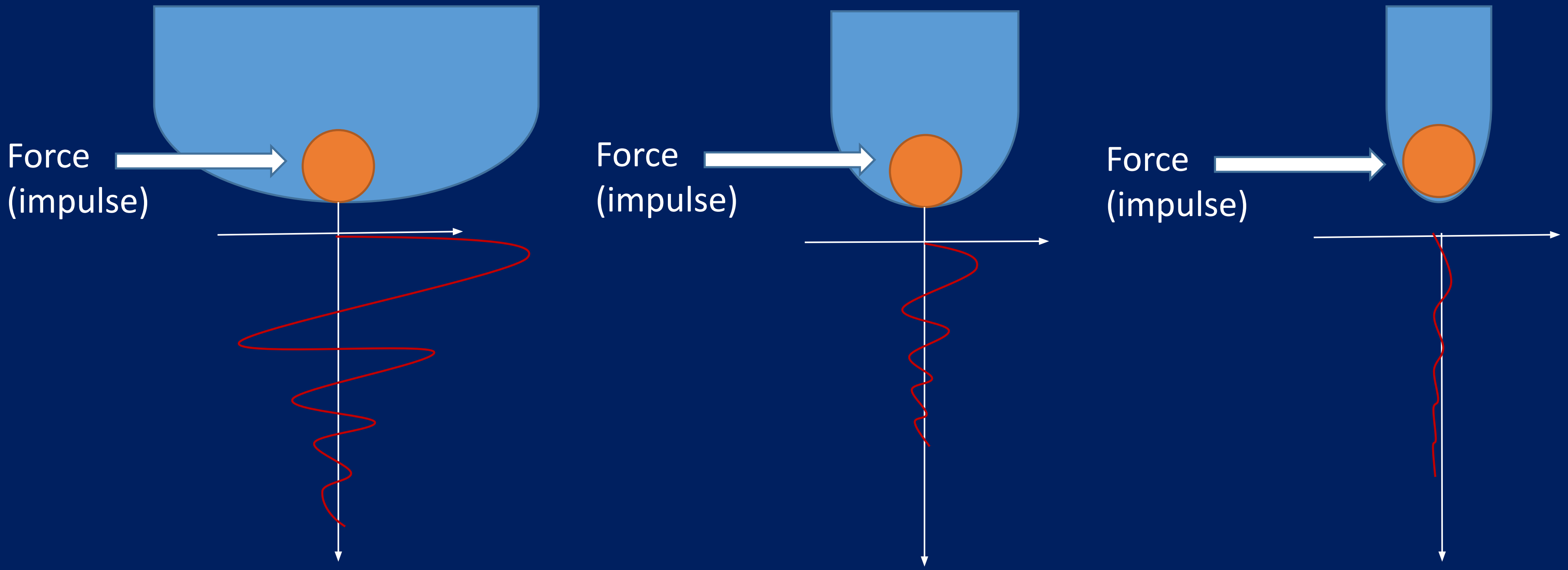
Time Response:
a. Transient
b. Steady State
Frequency Response:
a. Body Plot
Stability:
a. Routh-Hurwitz
b. Nyquist

DESIGN

Controller
Root – Locus
Lead – Lag compensator with Bode Plot
Lead – Lag compensator with Root Lotus


NEVER SAY NEVER, BECAUSE LIMITS,
LIKE FEARS, ARE OFTEN JUST AN
ILLUSION
vrawdopest.tumblr
Michael Jordan
BULLS
23

System changes over the time and has to handle variation of system parameters.
STABILITY MARGIN = ROBUST DESIGN



Stable

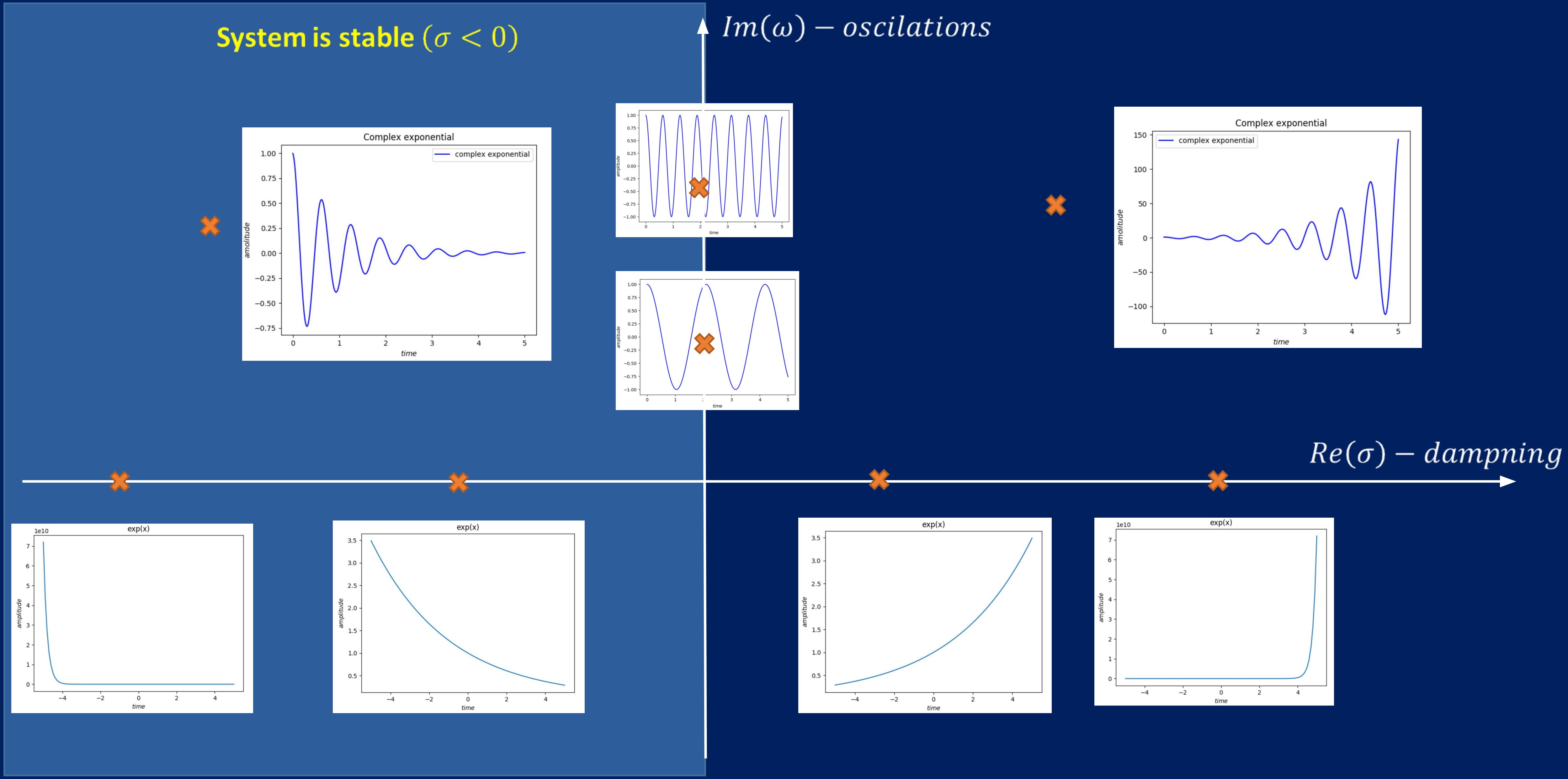
More Stable

Stiff – Too Stable

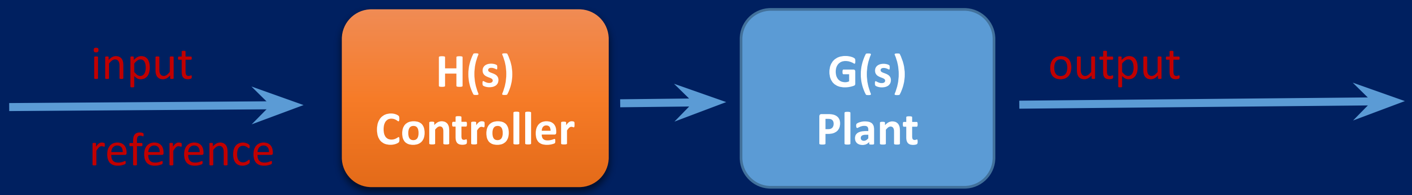
It does not respond to disturbances – OK
It does not respond to actual command – **NOT OK**

Analysis system response (location of poles)

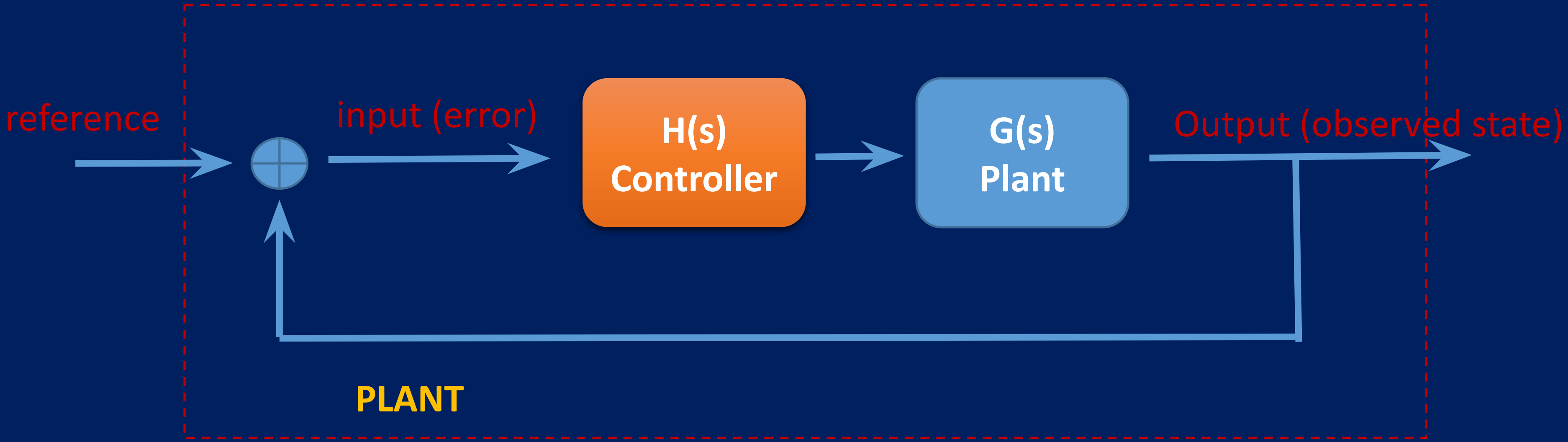
$$\textbf{System transfer function} = \frac{\textbf{zeros}}{\textbf{poles}}$$



OPEN LOOP

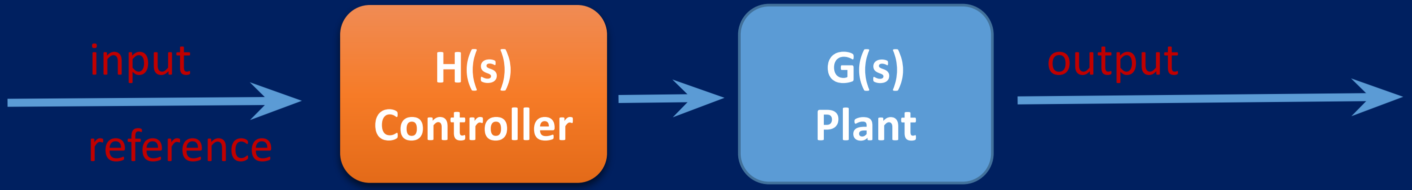


CLOSED LOOP



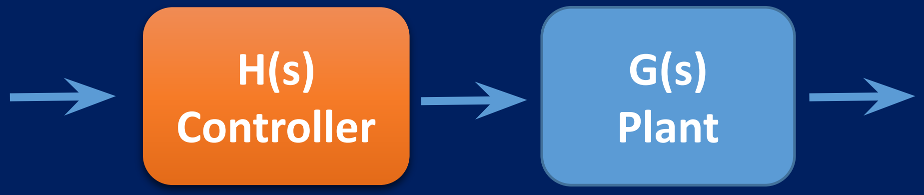
Reference does not depend on the output so we can consider as a OPEN system again

OPEN LOOP



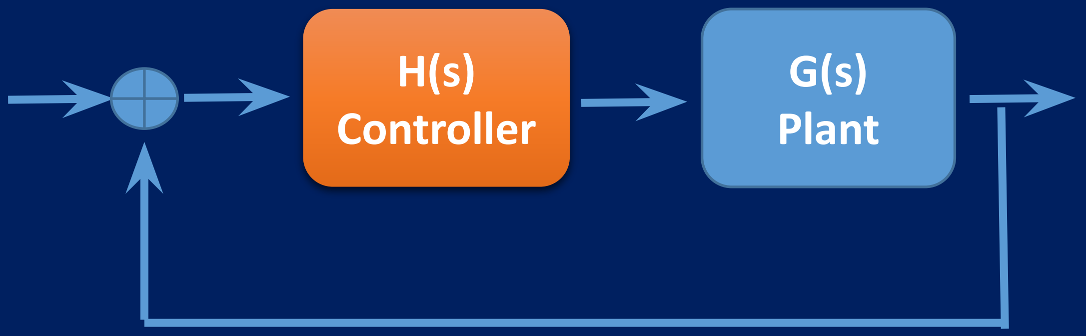
Stability analysis for CLOSE system can be consider as an OPEN system rewriting the system equations (transfer function)!!! (we use the same techniques)

OPEN LOOP



$$G(s) = \frac{N(s)}{D(s)} \quad \begin{matrix} -nominator \ - ZEROS \\ -denominator \ - POLES \end{matrix}$$

CLOSE LOOP



$$TF(s) = \frac{H(s) \cdot G(s)}{\boxed{1 + H(s) \cdot G(s)}}$$

Characteristic equation

$$D(s) = \frac{1}{(s+1) \cdot (s+2)}$$

H(s) – design by engineer

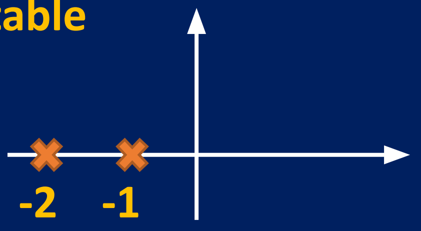
G(s) – represent dynamics of the system

$D(s) - we\ analyse\ the\ roots\ (poles)\ to\ check\ stability$

$$1 + H(s) \cdot G(s)$$

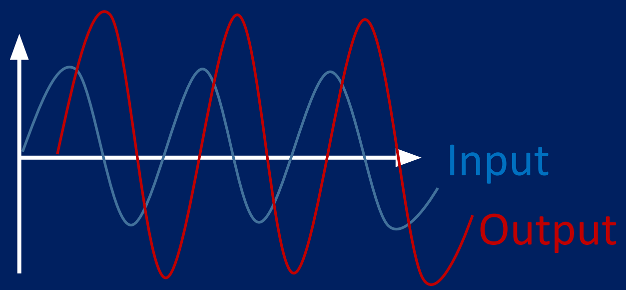
- In order to divide by 0 we need to secure that: 1+H(s)G(s) = 0 therefore we are looking for location of zeros.

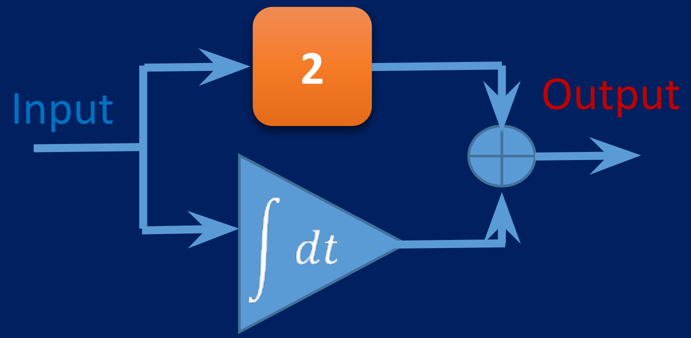
Stable



BODE PLOT

In LTI systems the f(t) = sin(t) DOES NOT the shape and frequency (only one wave form). The only differences are amplitude and phase.





$$Input = \sin\left(\frac{1}{2}t\right);\ \omega = \frac{1}{2}{rad}/{s}$$

$$Output = 2\sin\left(\frac{1}{2}t\right) + \int \sin\left(\frac{1}{2}t\right)dt =$$

$$= 2\sin\left(\frac{1}{2}t\right) - 2\cos\left(\frac{1}{2}t\right)$$

$$a = 2, b = -2$$

$$\cong 2.83\sin\left(\frac{1}{2}t - 0.785\right)$$

Amplitude Phase

Frequency the same

$$\text{a}\sin x + \text{b}\sin x = \sqrt{a^2+b^2}\sin(x+\theta)$$

$$\theta = \text{atan}\left(\frac{b}{a}\right)$$

We would like to check how the system respond for different frequencies!

$$2.83\sin\left(\frac{1}{2}t-0.785\right)$$

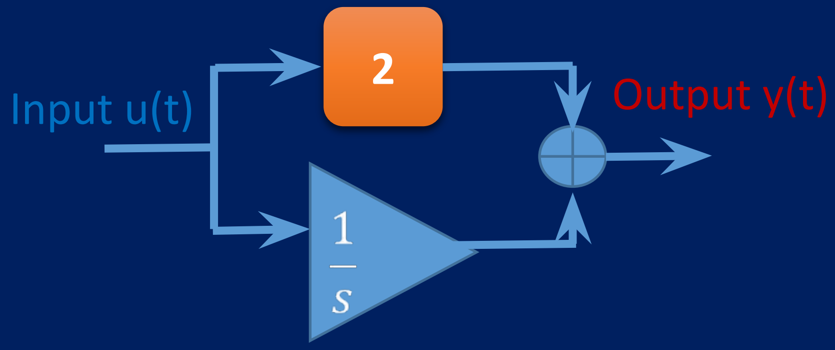
$$1\text{dB}=20\,log_{10}\frac{A\_out}{A\_in}\Rightarrow$$

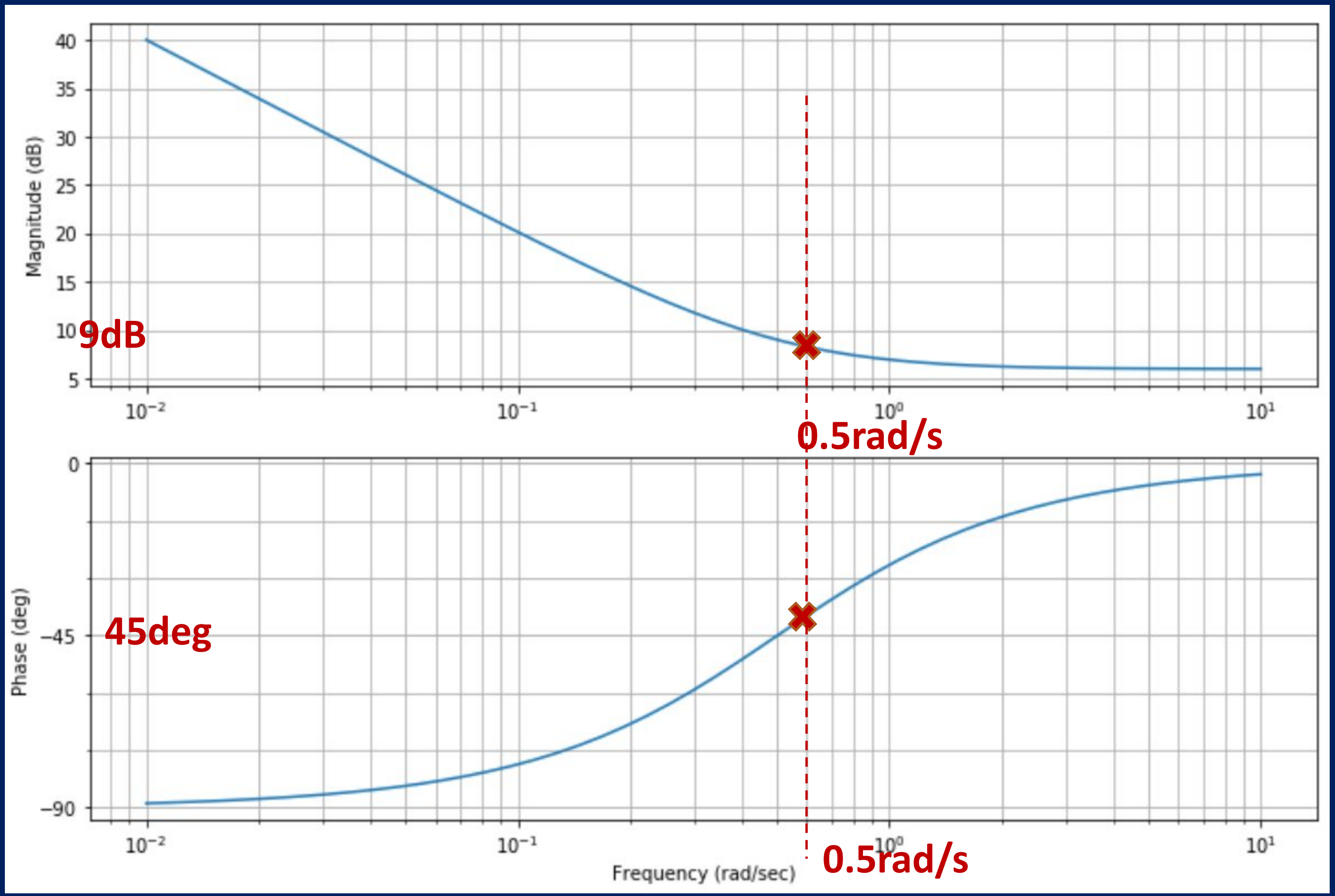
$$20\,log_{10}\frac{2.83}{1}\cong 9\text{dB}$$

$$\frac{180°}{\pi}\cdot 0.785=45°$$

We consider steady state system so $s=\sigma+j\omega$

$\sigma=0\Rightarrow\quad s=j\omega$





$$amplitude\ (GAIN)=\sqrt{Real^2+Img^2}$$

$$phase(SHIFT)=\text{atan2}\left(\frac{Img}{Real}\right)$$

$$s=\sigma+j\omega$$

$$y(t)=\left(2+\frac{1}{s}\right)u(t)\Rightarrow\frac{y(t)}{u(t)}=\frac{2s+1}{s}\Rightarrow\frac{2j\omega+1}{j\omega}=2-\frac{1}{\omega}j$$

$\omega\to\infty\Rightarrow amp\to 2\Rightarrow$

$\omega\to\infty\Rightarrow phase\to 0$

$\omega\to 0\Rightarrow phase\to 90deg$

BODE PLOT – GAIN

K = 2

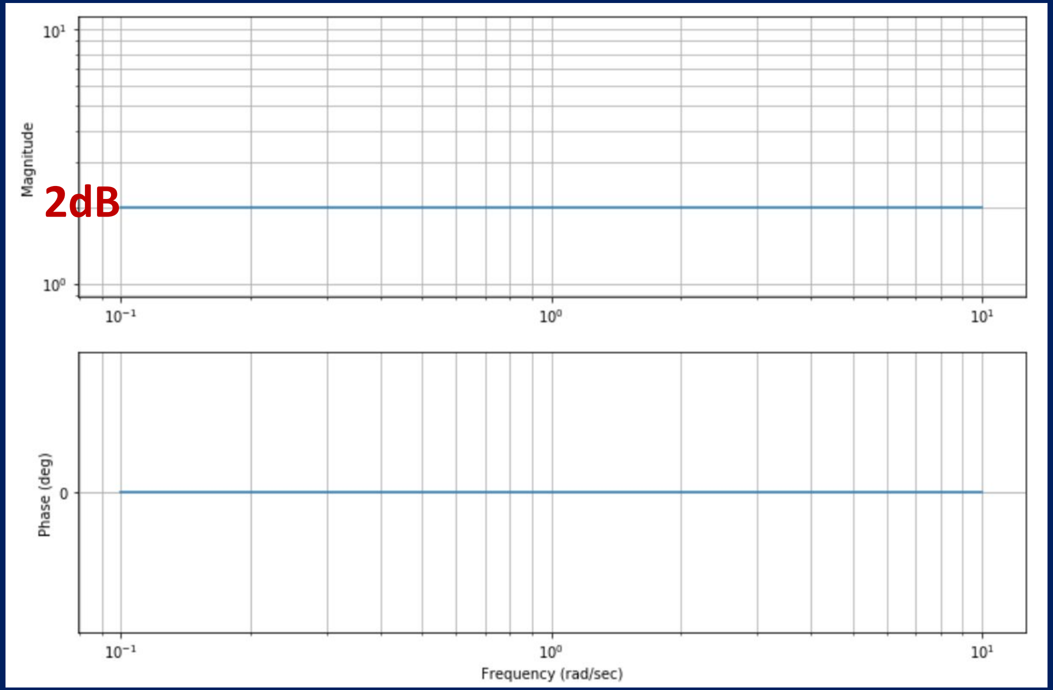
$$\frac{y(t)}{u(t)} = 2$$

$$amplitude\ (GAIN) = \sqrt{\boldsymbol{Real}^2 + \boldsymbol{Img}^2} = 2$$

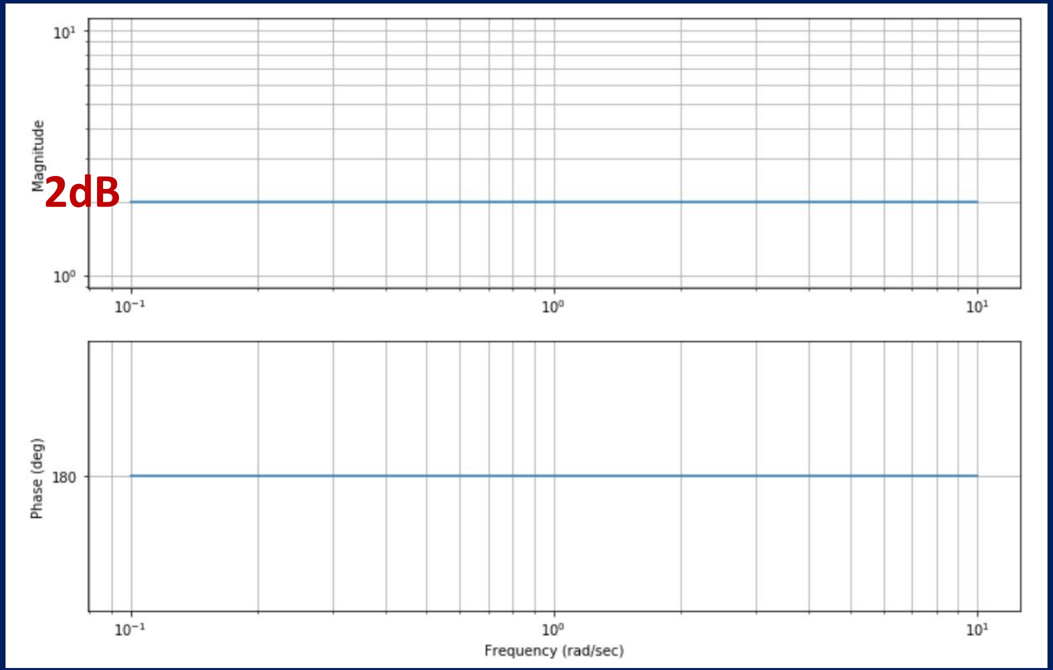
$$phase(SHIFT) = \operatorname{atan2}\left(\frac{\mathbf{0}}{\boldsymbol{K}}\right) = 0°\ for\ K > 0$$

$$phase(SHIFT) = \operatorname{atan2}\left(\frac{\mathbf{0}}{\boldsymbol{K}}\right) = 180°\ for\ K < 0$$

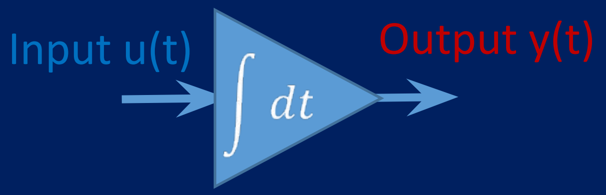
$K > 0$



$K < 0$



BODE PLOT – INTEGRAL (Phase LAG) - poles



$$\int dt$$

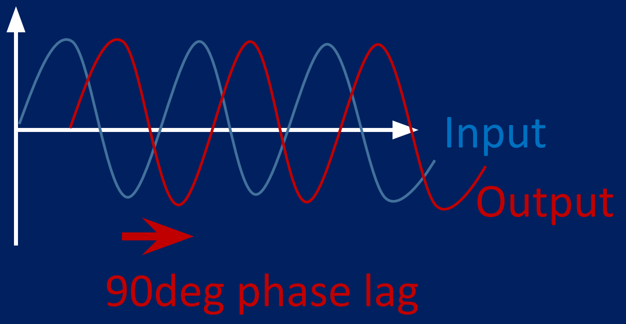
$$Input = \sin(\omega t)$$

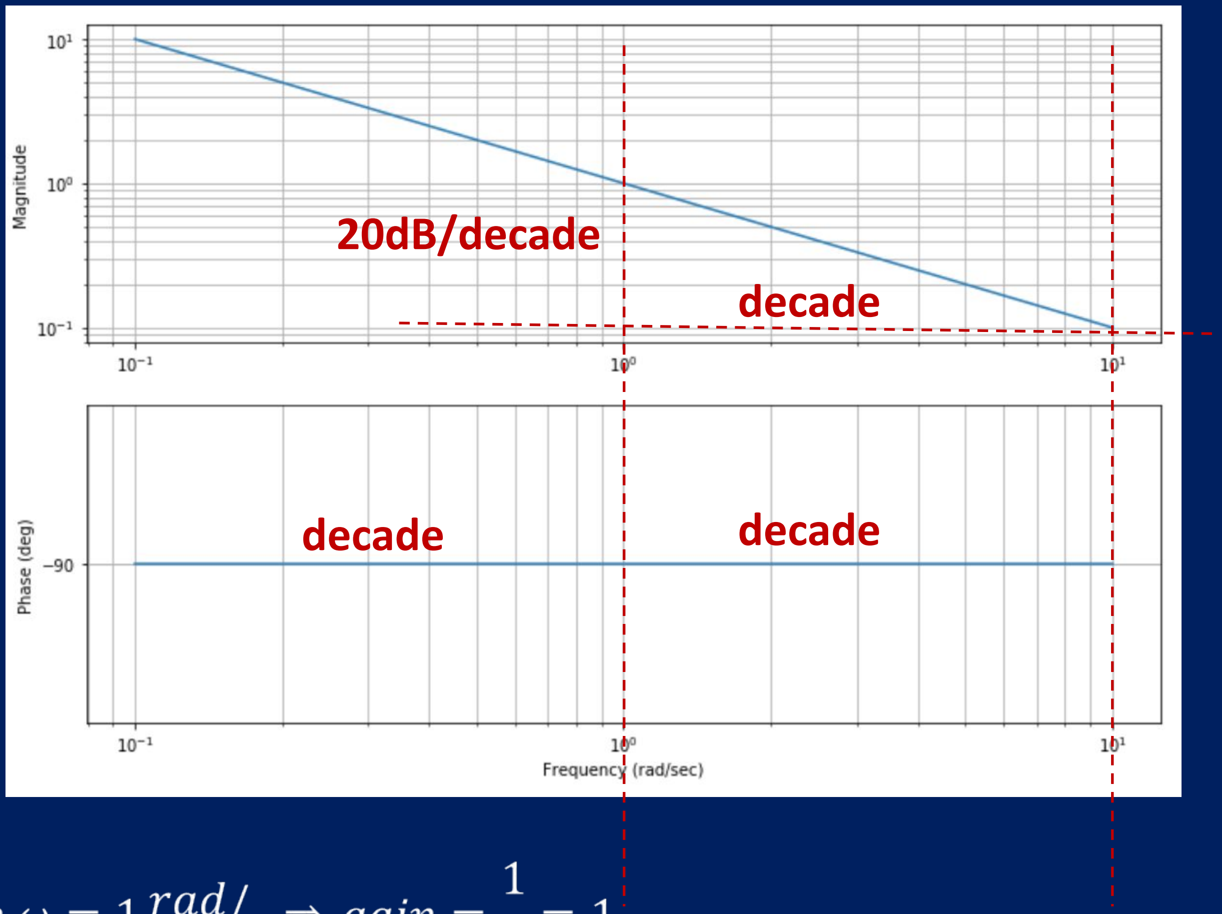
$$Output = \int \sin(\omega t)\,dt = -\frac{1}{\omega}\cos(\omega t)$$

$$\frac{y(t)}{u(t)} = \frac{1}{s} = -\frac{1}{\omega}j$$

$$amplitude\ (GAIN) = \sqrt{\boldsymbol{Real}^2 + \boldsymbol{Img}^2} = \frac{1}{\omega}$$

$$phase(SHIFT) = \operatorname{atan2}\left(\frac{\boldsymbol{1}}{\frac{\boldsymbol{\omega}}{\boldsymbol{0}}}\right) = -90°$$





$$when\ \omega = 1\,{}^{rad}/_{s} \Rightarrow gain = \frac{1}{1} = 1$$

$$20\log_{10} 1 = 0$$

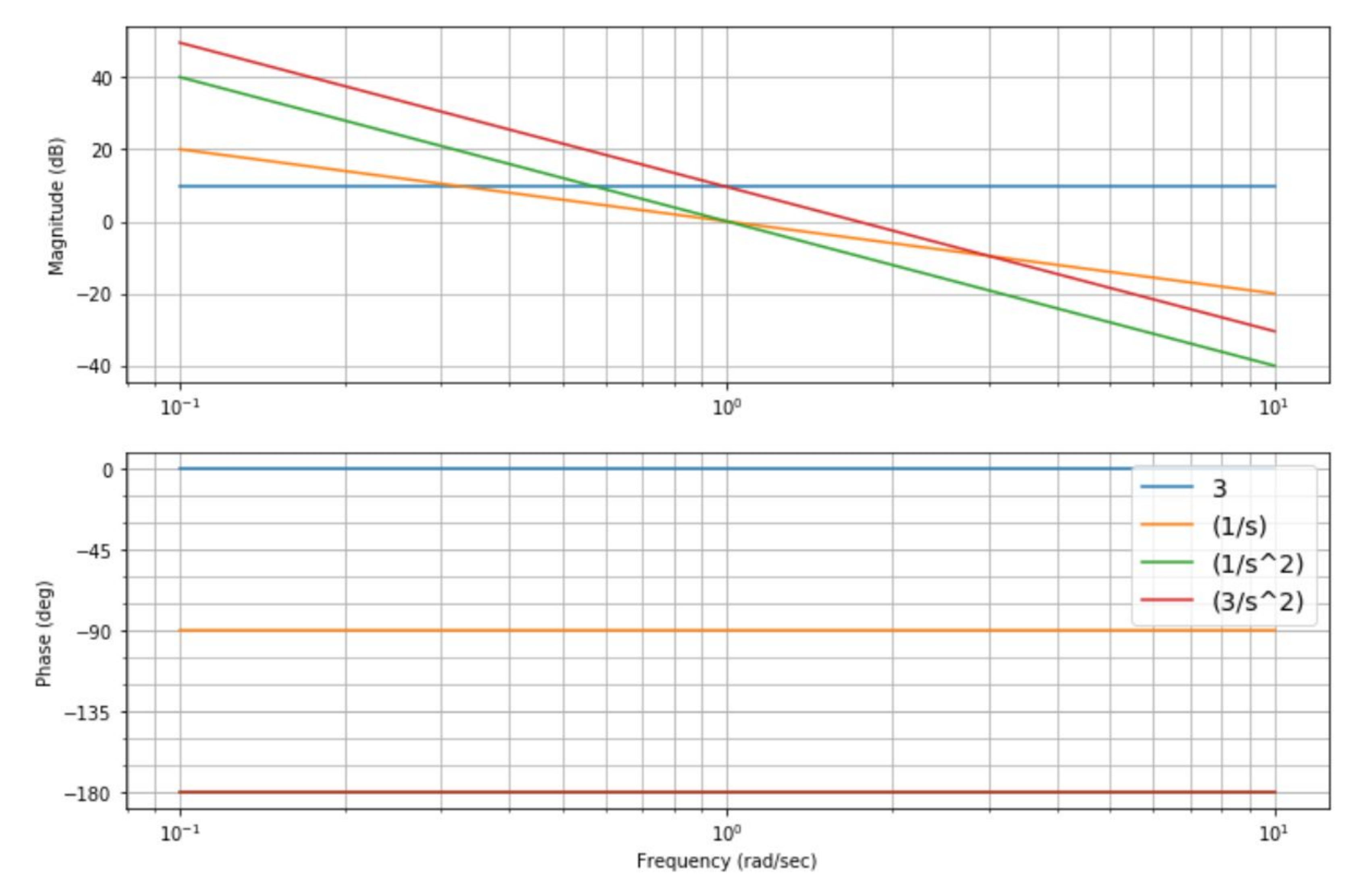
$$when\ \omega = 10\,{}^{rad}/_{s}\,(decade) \Rightarrow gain = \frac{1}{10} = 0.1$$

$$20\log_{10} 1 = -20\,dB$$

$$\frac{y(t)}{u(t)} = \frac{3}{s^2} = 3 \cdot \frac{1}{s} \cdot \frac{1}{s}$$

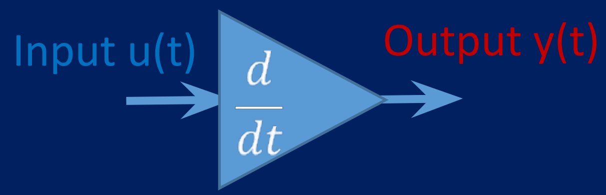
$$\frac{1}{s} \Rightarrow \text{gain drops} \frac{20\text{db}}{\text{decade}}; \text{phase} = -90°$$

$$\textit{In dB domain we sum each individual response } 3, \frac{1}{s}, \frac{1}{s}$$



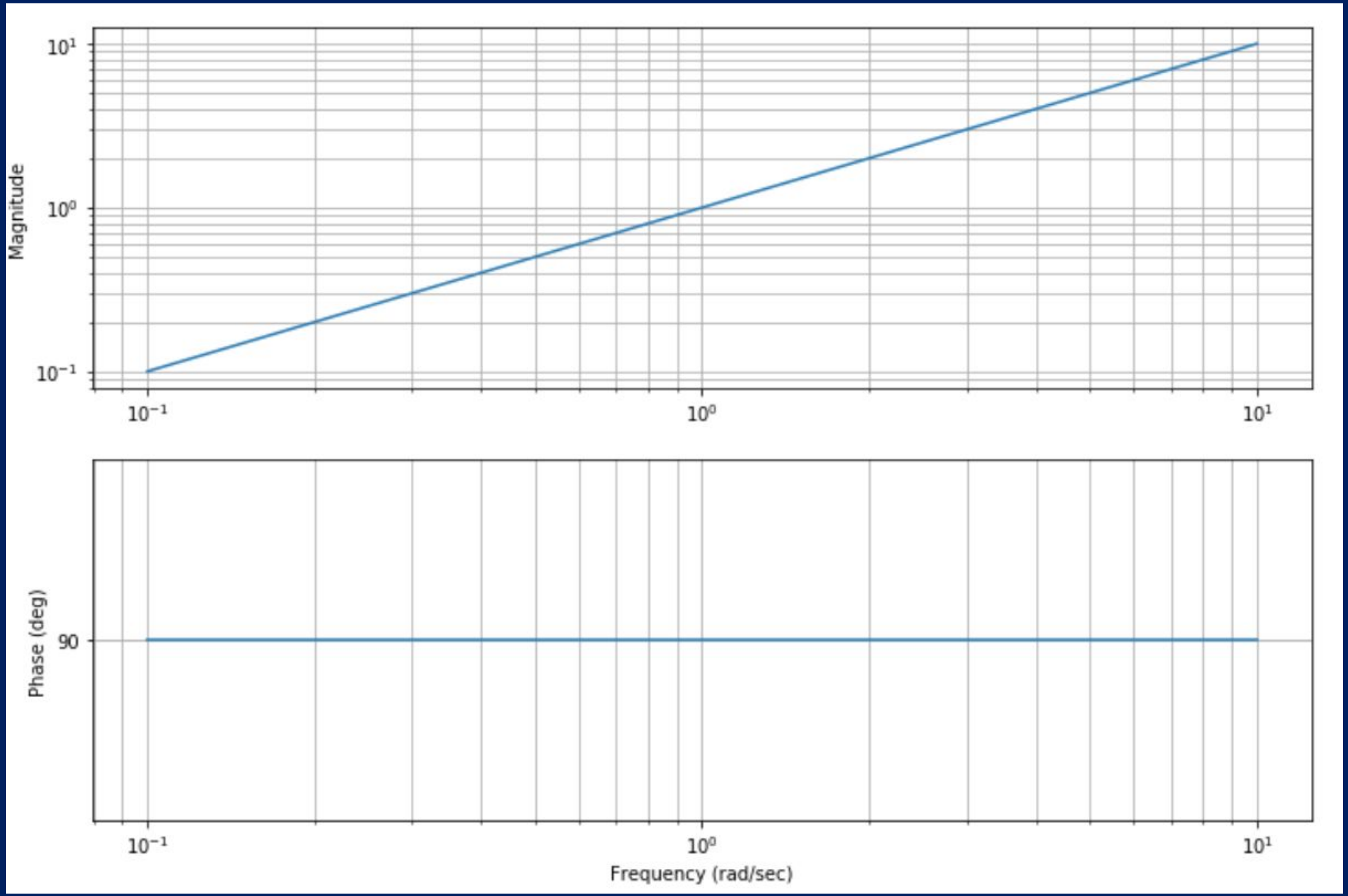
BODE PLOT – DERIVATIVE (phase LEAD) - zeros

$$\frac{d}{dt} = s$$



$$Input = \sin(\omega t)$$

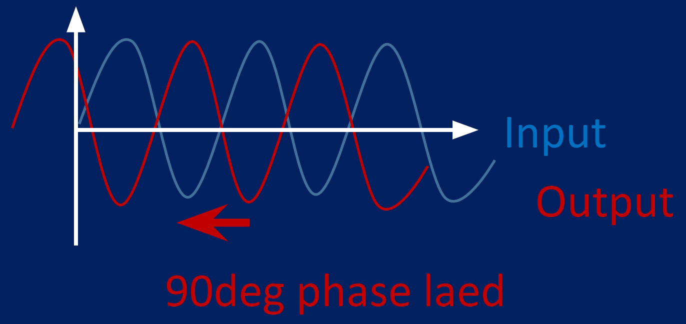
$$Output = \frac{d}{dt}\sin(\omega t) = \omega\cos(\omega t)$$



$$\frac{y(t)}{u(t)} = s = j\omega$$

$$amplitude\ (GAIN) = \sqrt{\boldsymbol{Real}^2 + \boldsymbol{Img}^2} = \omega$$

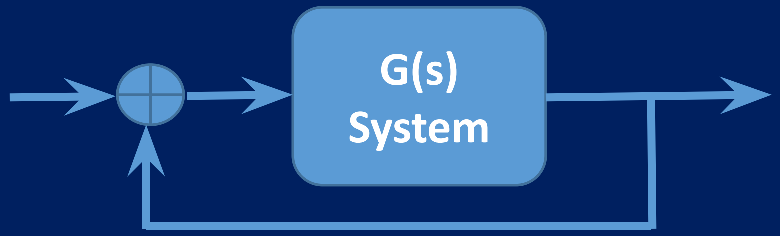
$$phase(SHIFT) = \operatorname{atan} 2(\omega) = 90°$$



OPEN LOOP (OL)

$$G(s) = \frac{Z(s)}{P(s)}$$

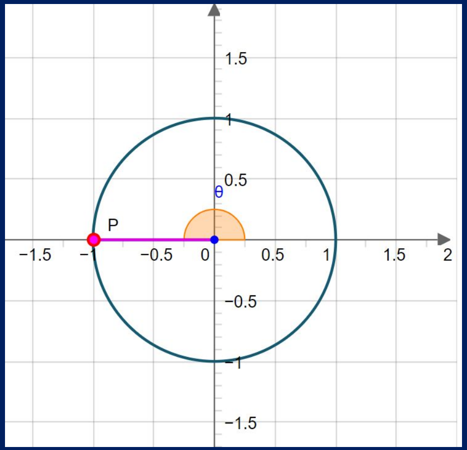
CLOSE LOOP (CL)



$$\frac{G(s)}{1 + G(s)}$$

$$\frac{G(s)}{1 + G(s)} = \frac{\frac{Z(s)}{P(s)}}{1 + \frac{Z(s)}{P(s)}} = \frac{\frac{Z(s)}{P(s)}}{\frac{P(s) + Z(s)}{P(s)}} = \frac{Z(s)}{Z(s) + P(s)}$$

It can be very difficult to design CL system so normally we assess the OL
By use of Bode, Nyquist, Locus-Root to estimate the stability and performance of CL.



$$G(s) = \frac{Z(s)}{P(s)}$$

1. CL system will be UNSTABLE if poles P(s) of G(s) are in the right half plane.

2. CL system will be UNSTABLE if the gain of the system is infinity (response will grow unbounded).

$$\frac{G(s)}{1+G(s)} = \infty$$

It will happen of 1+G(s) = 0 => **G(s) = -1 ---- WE HAVE TO AVOID THIS**

$$A \cdot e^{(\sigma + j\omega)\cdot t} = A \cdot \cos(\omega t + \sigma t) + j \cdot A \cdot \sin(\omega t + \sigma t) \Rightarrow gain \cdot [\cos(phase) + j \cdot \sin(phase)]$$
$$= 1 \cdot [\cos(-180°) + j \cdot \sin(-180°) = -1 - 0j]$$

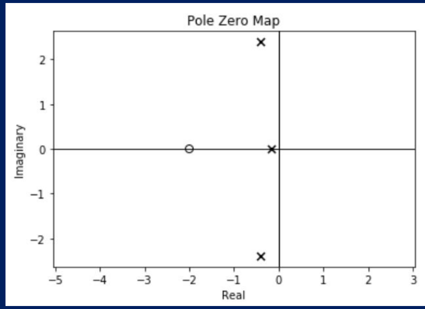
It means that if we keep our open loop system away from -1 so our close-loop system will be stable.

-1 => Gain = 1 => 0dB => $20\log_{10} 1 = 0$

-1 => Phase = -180deg => because it flips the input upside down

If one frequency produces 0dB AND -180deg phase so the close loop is unstable.

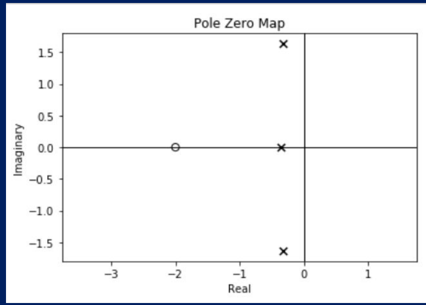
OPEN LOOP

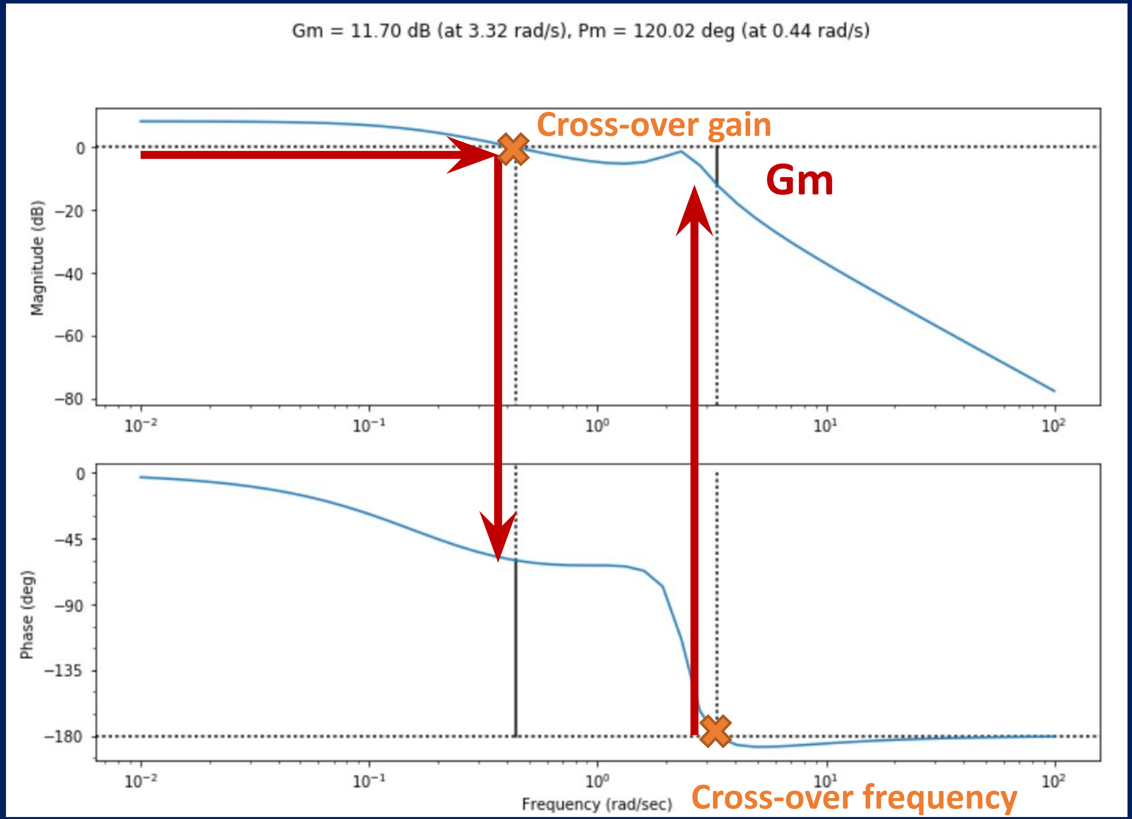


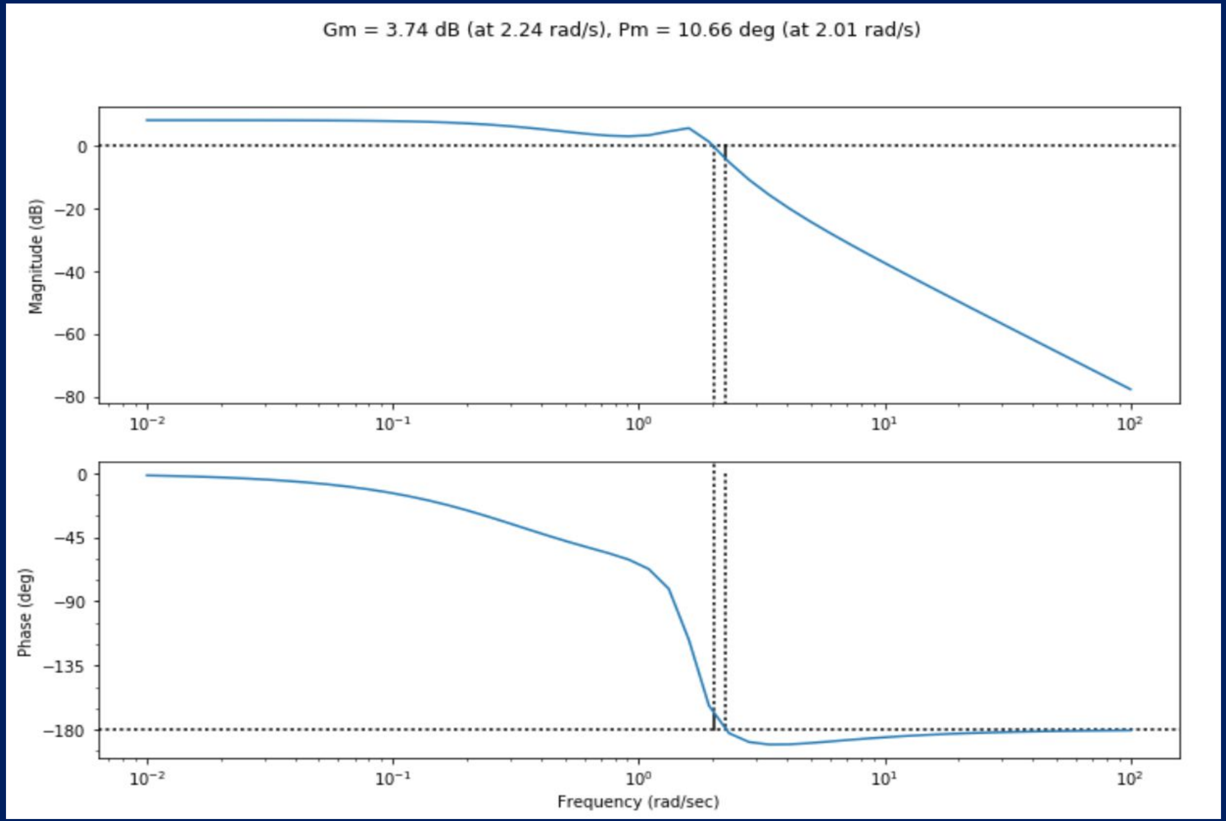
By reducing a little G(s) WE GO ALMOST TO UNSTABLE => WE DO NOT MARGIN ALMOST AT ALL!!!

$$\frac{1.3s + 2.6}{s^3 + s^2 + 6s + 1} \Rightarrow \frac{1.3s + 2.6}{s^3 + s^2 + 3s + 1}$$

OPEN LOOP





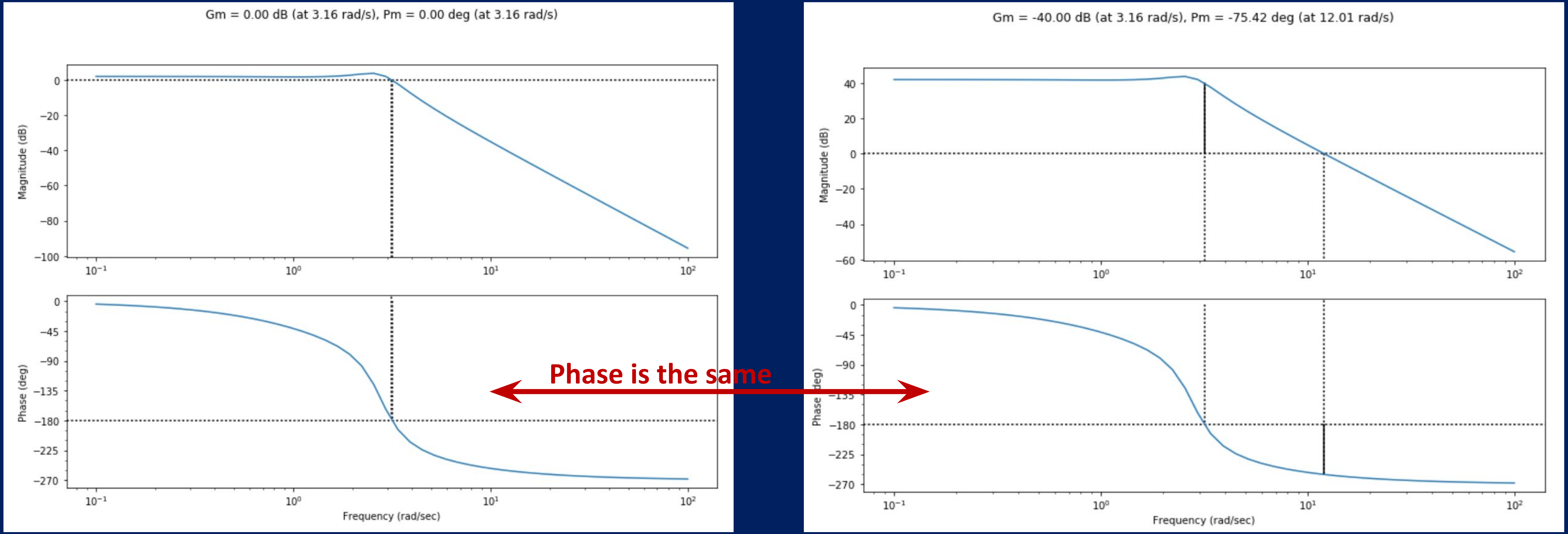


The gain margin Gm is defined as the change in open-loop gain required to make the closed-loop system unstable. Systems with greater gain margins can withstand greater changes in system parameters before becoming unstable in closed-loop.

The phase margin Pm is defined as the change in open-loop phase shift required to make the closed-loop system unstable. The phase margin also measures the system's tolerance to time delay.

$$\frac{50}{s^3 + 9s^2 + 30s + 40} \Rightarrow \frac{\mathbf{100} * 50}{s^3 + 9s^2 + 30s + 40}$$

By increasing a G(s) WE INCREASE STABILITY => WE HAVE GOOD MARGIN !!! **(Phase is the same)**

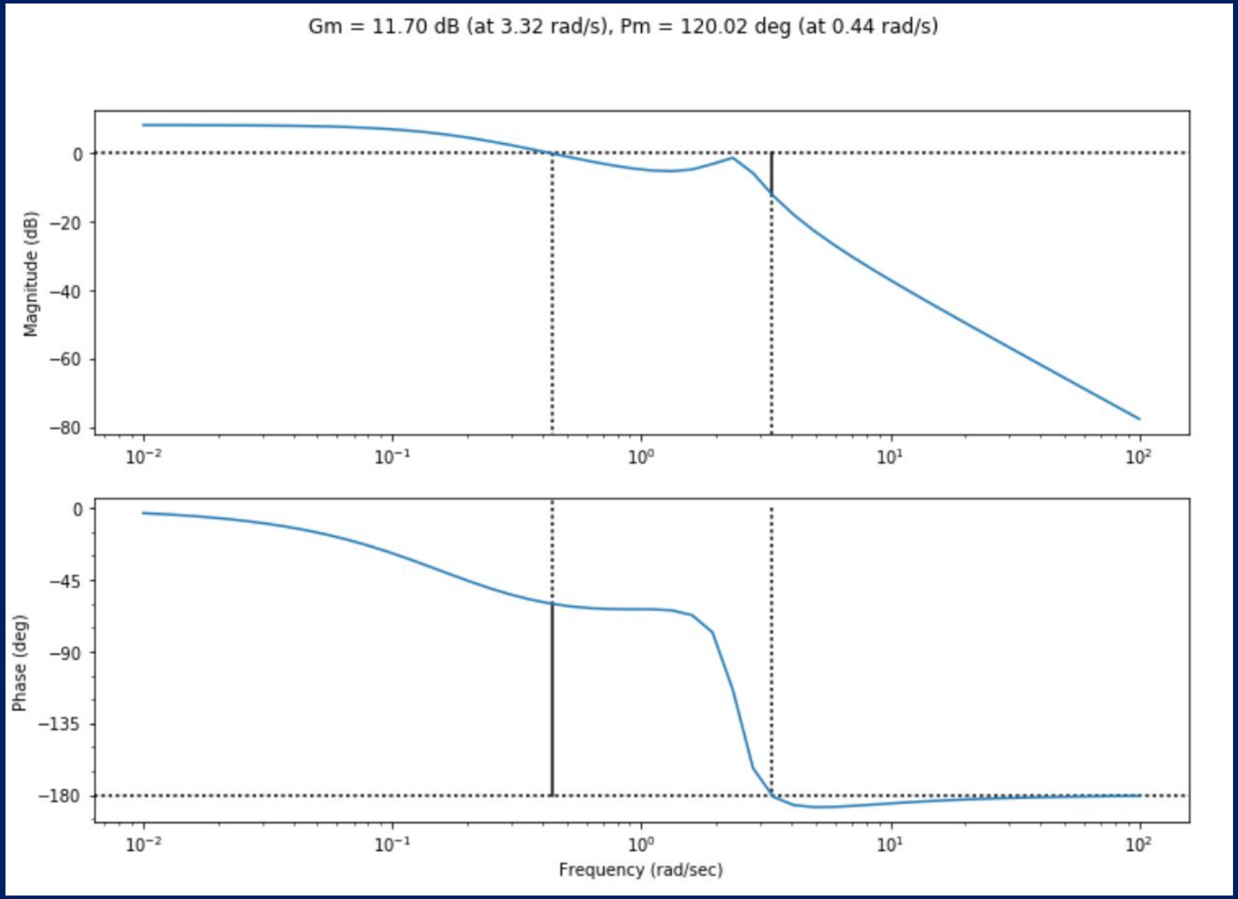


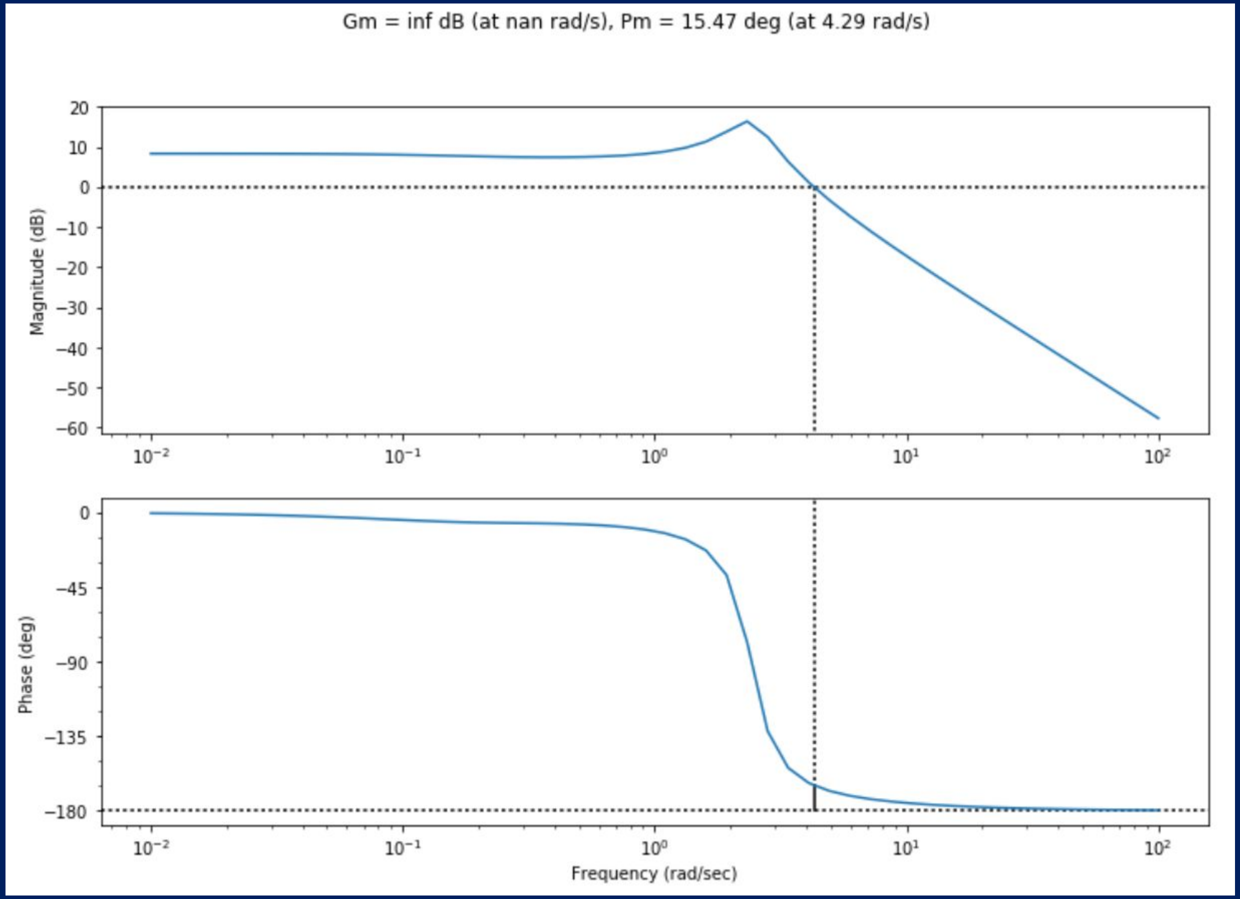
By increasing a little G(s) WE INCREASE GAIN MARGIN TO INFINITY =>

Gain margin (Gm) is infinity because phase does not cross 180deg

So we can give as much as we want.

$$\frac{1.3s + 2.6}{s^3 + s^2 + 6s + 1} \Rightarrow \frac{10.3s + 2.6}{s^3 + s^2 + 6s + 1}$$





NYQUIST DIAGRAM. STABILITY

For LTI

$u(t) = D\sin(\omega t)$

$y(t) = DA\sin(\omega t + \theta)$

$$A = |G(j\omega)|$$

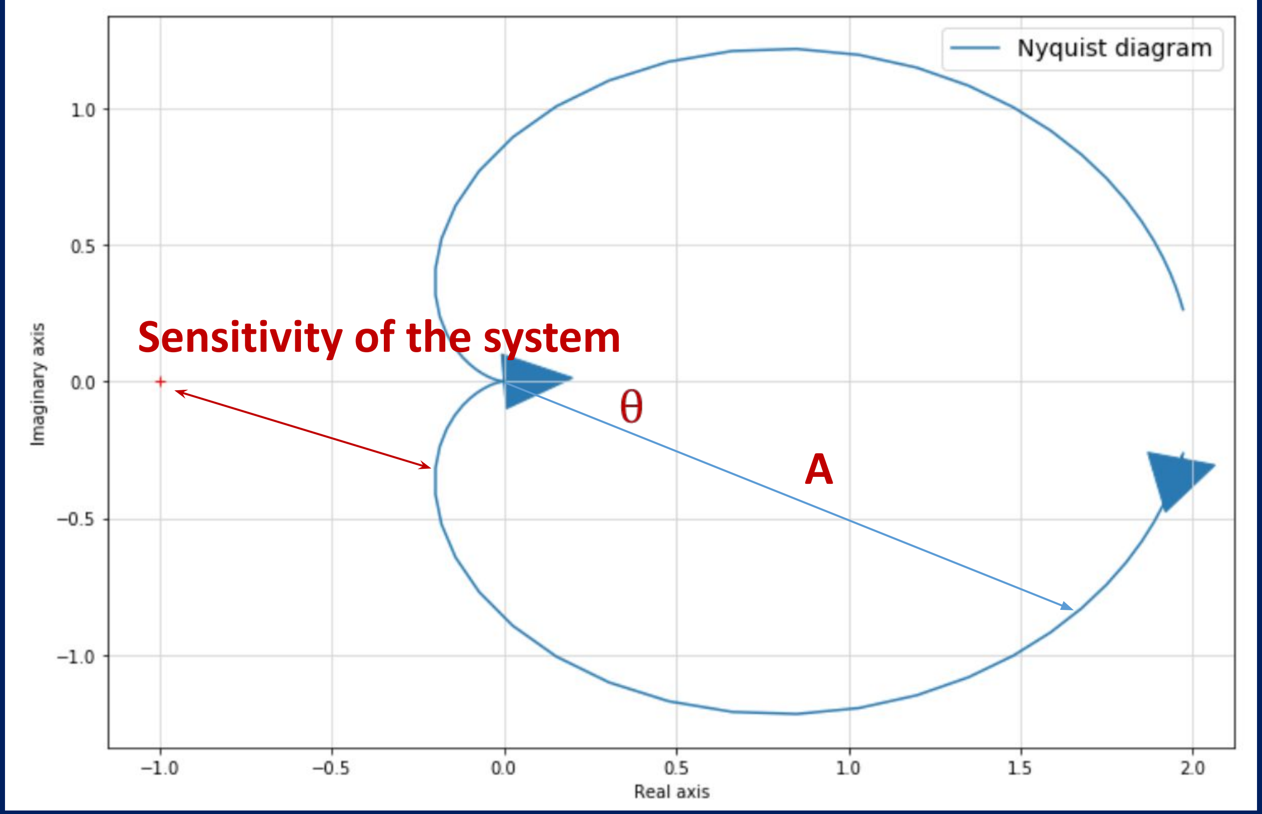
$$\theta = \arg(G(j\omega))$$

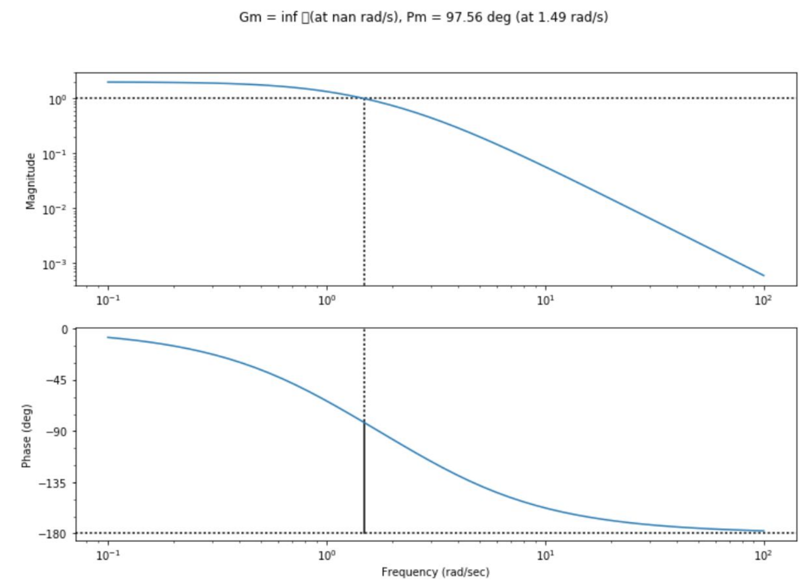
Nyquist diagram is a plot of $G(j\omega)$, in the complex plane.

$$G(s) = \frac{6}{s^2 + 4s + 1}$$

$$A \cdot e^{(\sigma + j\omega)\cdot t} = A \cdot \cos(\omega t + \sigma t) + j \cdot A \cdot \sin(\omega t + \sigma t)$$

	A		Complex
0	2	0	2
1	1.34	-63	0.6-1.2j
3	0.45	-116	-0.2-0.4j
10	0.06	-157	-0.05-0.02j





The Nyquist diagram allows us to predict the stability and performance of a closed-loop system by observing its open-loop behaviour.

The Nyquist criterion can be used for design purposes regardless of open-loop stability (remember that the Bode design methods assume that the system is stable in open-loop).

In order to estimate stability of CL system $\frac{G(s)}{1+G(s)}$, we analyse the OL system $G(s)=\frac{Z(s)}{P(s)}$

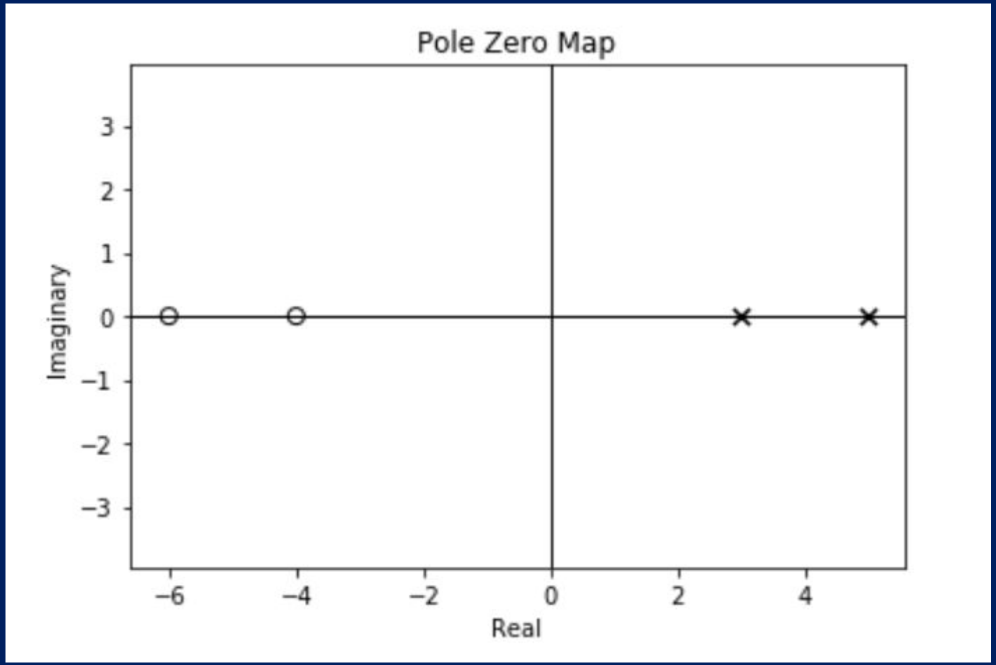
$$\frac{G(s)}{1+G(s)}=\frac{\frac{Z(s)}{P(s)}}{1+\frac{Z(s)}{P(s)}}=\frac{\frac{Z(s)}{P(s)}}{\frac{P(s)+Z(s)}{P(s)}}=\frac{Z(s)}{Z(s)+P(s)}$$

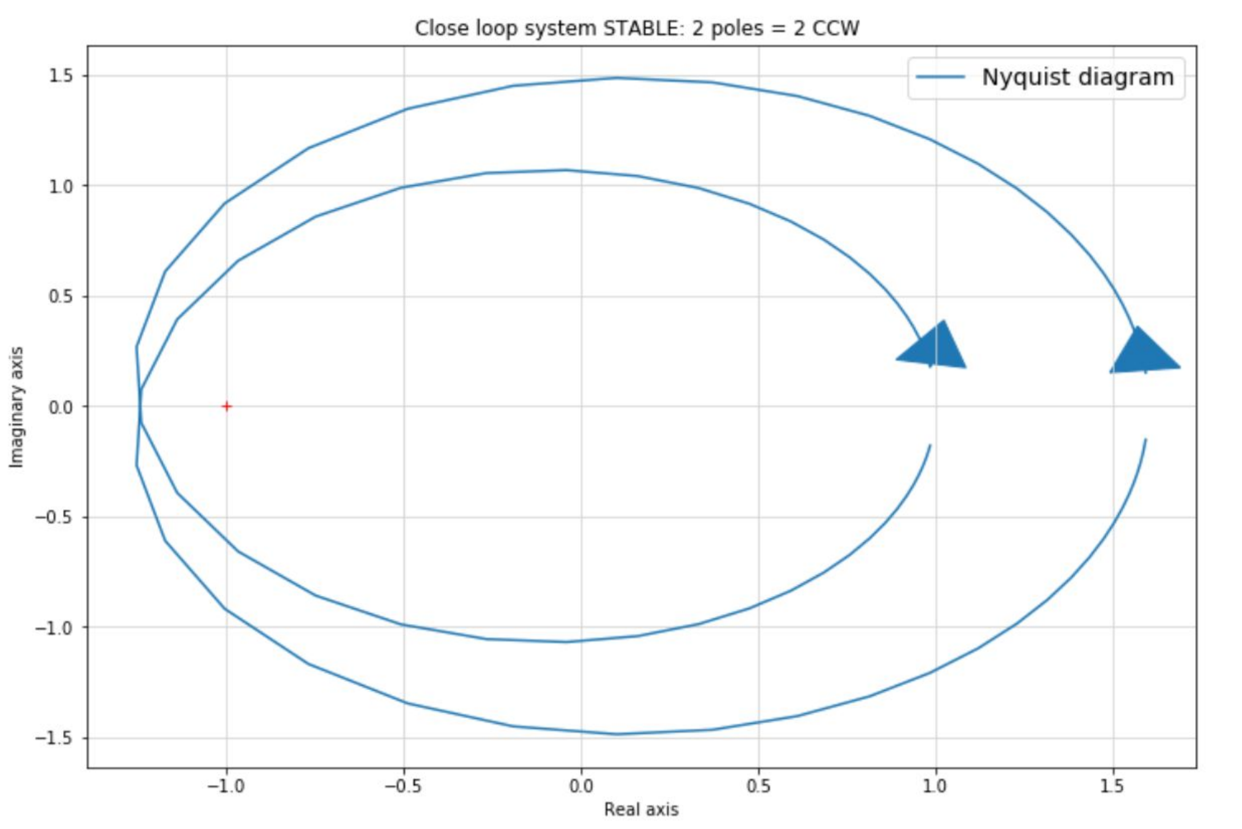
The Nyquist criterion : Z = N + P

Z - zeros of (1+ G(s)) in RHP = poles of CL transfer function so we want them in LHP
N - counter-clockwise encirclements of -1+j0
P - poles of G(s) in RHP

0 = N + P => P = -N

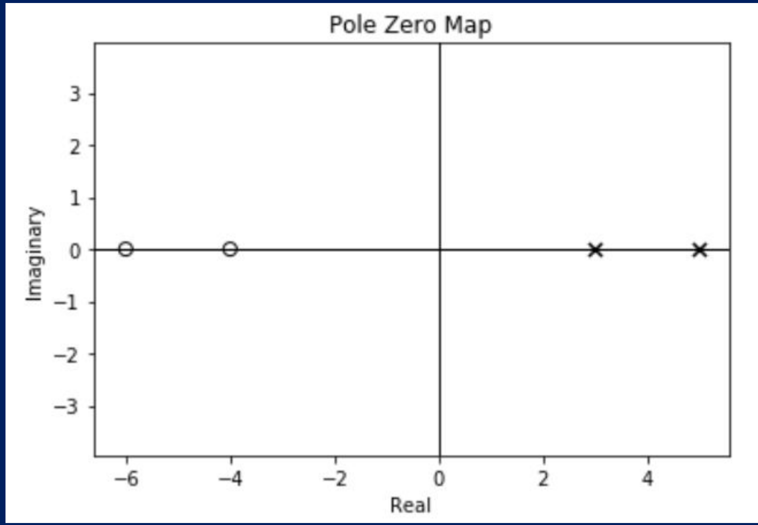
$$G(s) = \frac{s^2 + 10s + 24}{s^2 - 8s + 15}$$



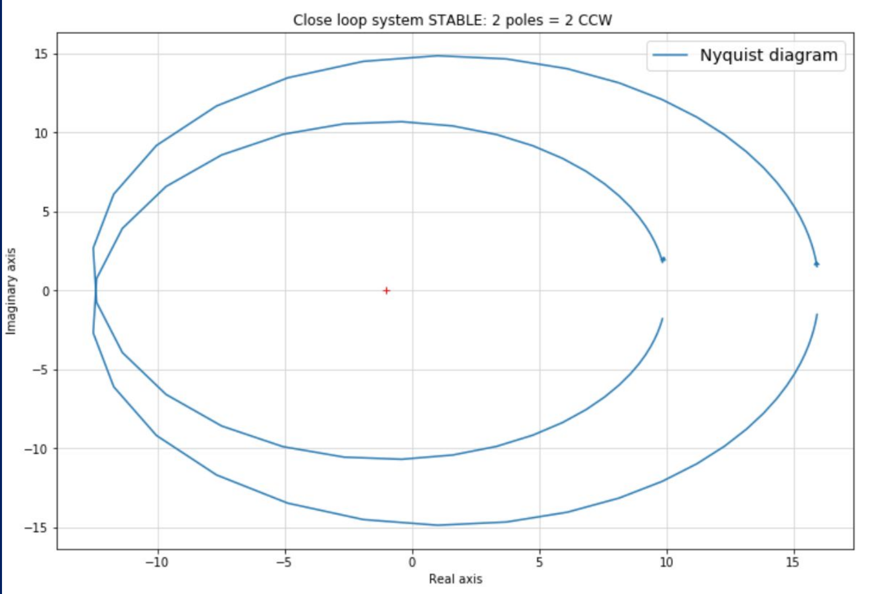


We increase gain = 10 $G(s) = \dfrac{10(s^2+10s+24)}{s^2-8s+15}$

Same as before

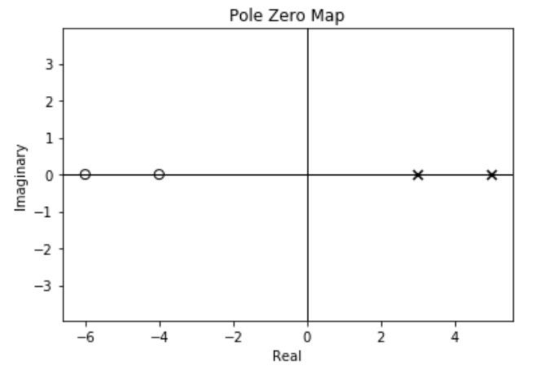


The diagram expanded => CL system will be stable no matter how much we increase the gain.

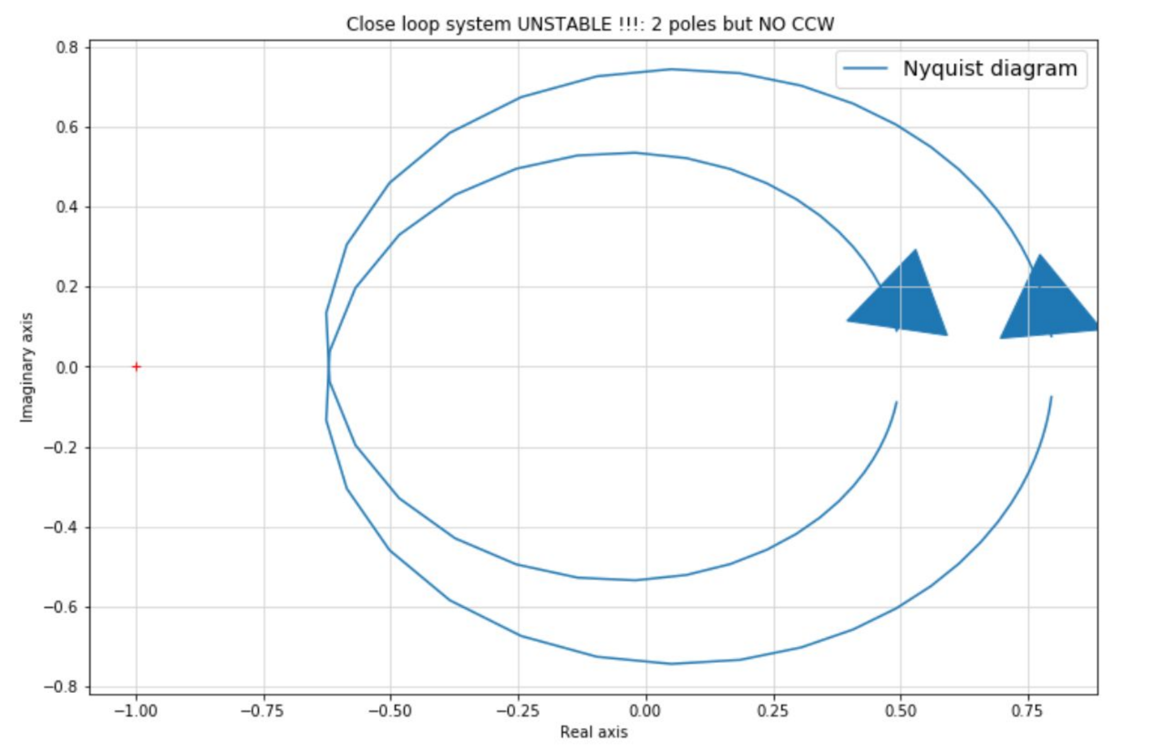


We decrease gain = 0.5 $G(s) = \dfrac{0.5(s^2+10s+24)}{s^2-8s+15}$

Same as before



SYSTEM UNSTABLE !!!!!

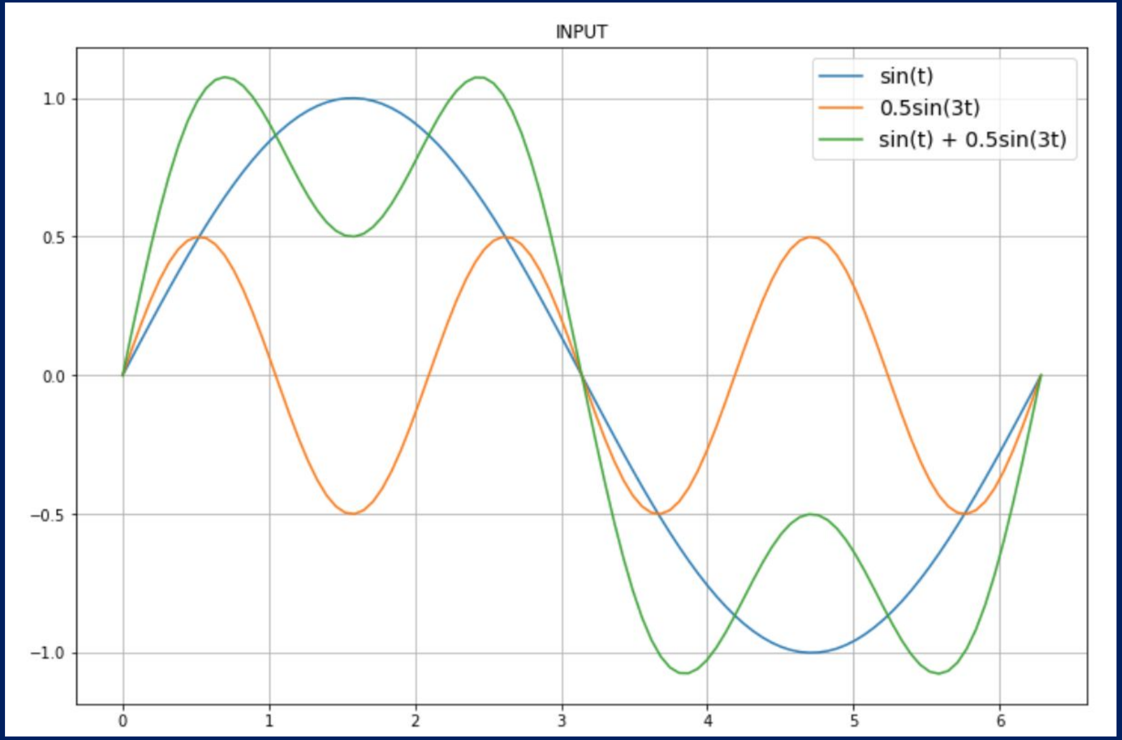


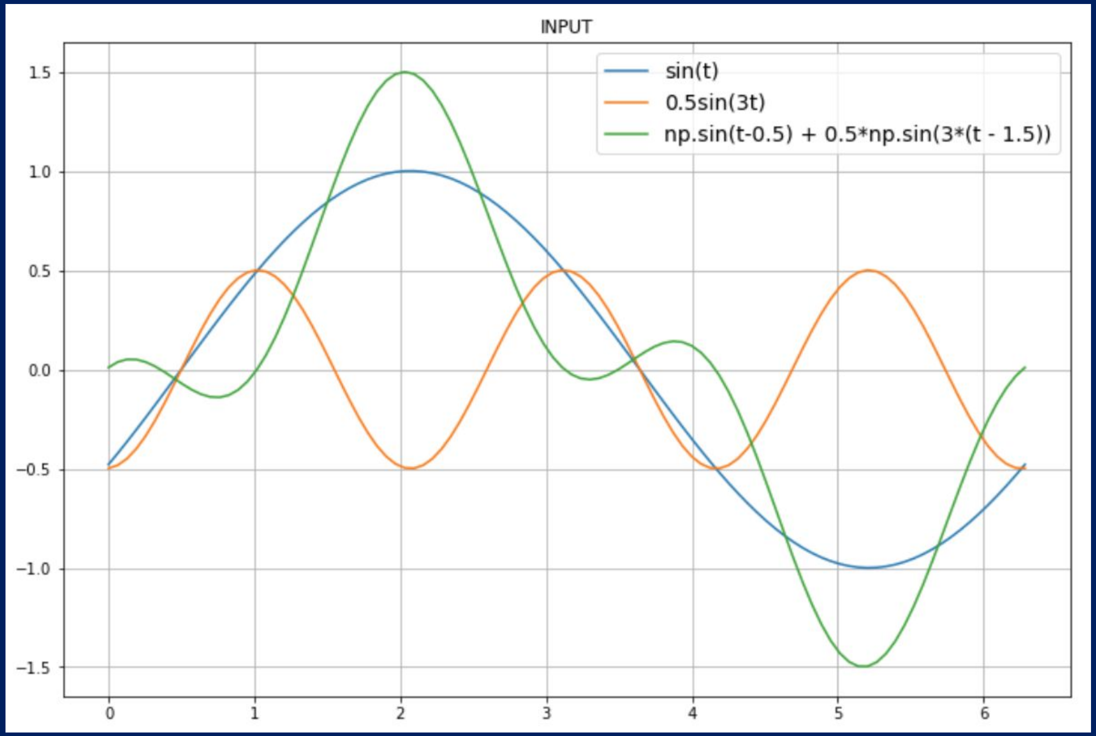
DELAY IN THE CONTROL SYSTEM. Bandwidth system limitation

Distorting delays = different delays for different frequencies of the signal

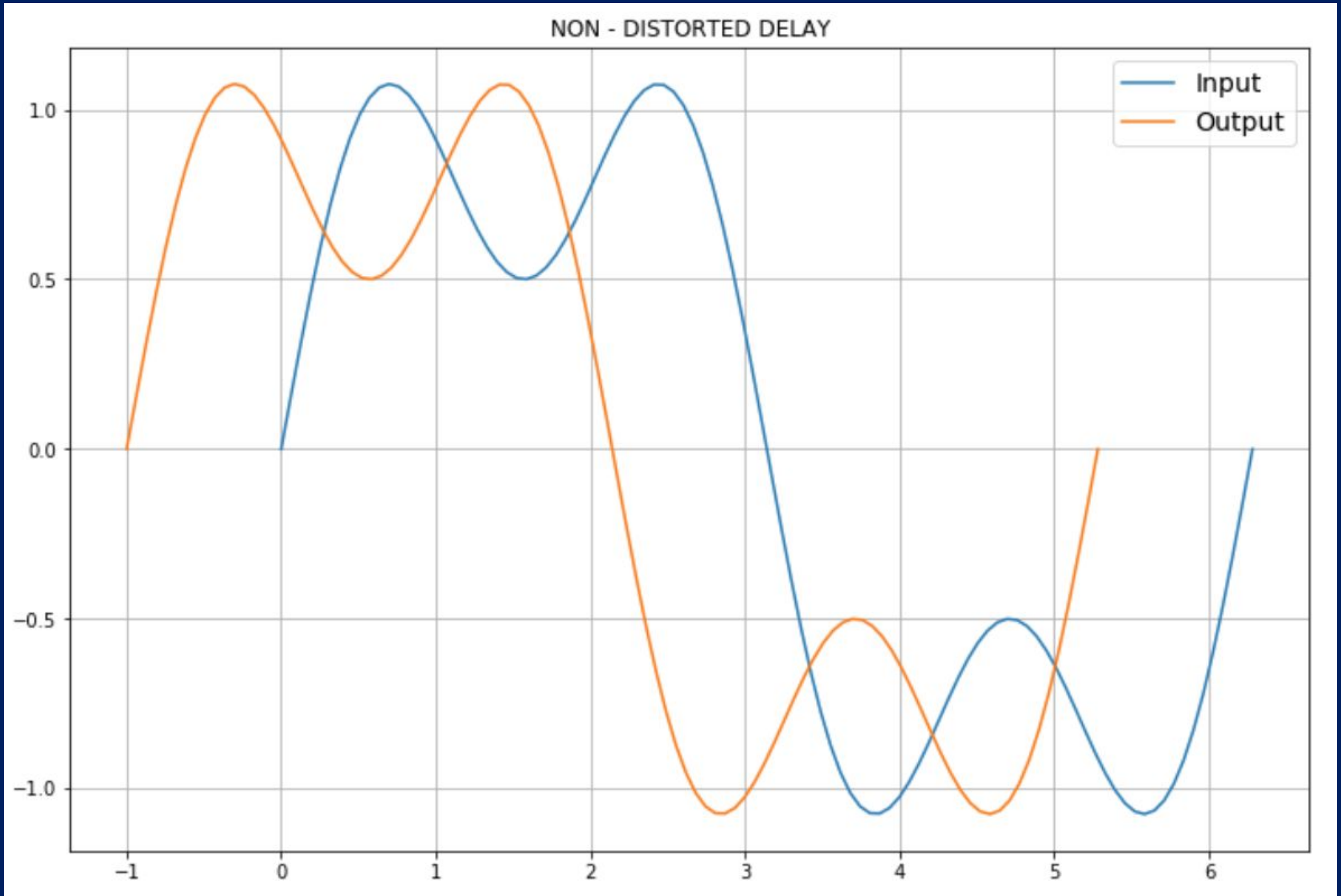
Non - distorting delays = affects entire signal equally

Distorting delays (each signal frequency is delay by different value so the signal is distorted – has other shape)





Non - distorting delays = affects entire signal equally (each frequency is delay equally)



Both of these delays exist in physical system and can cause the problems:

1. controller use the old information in order to determine the current output
2. the controller needs to predict the future how to output impact the system

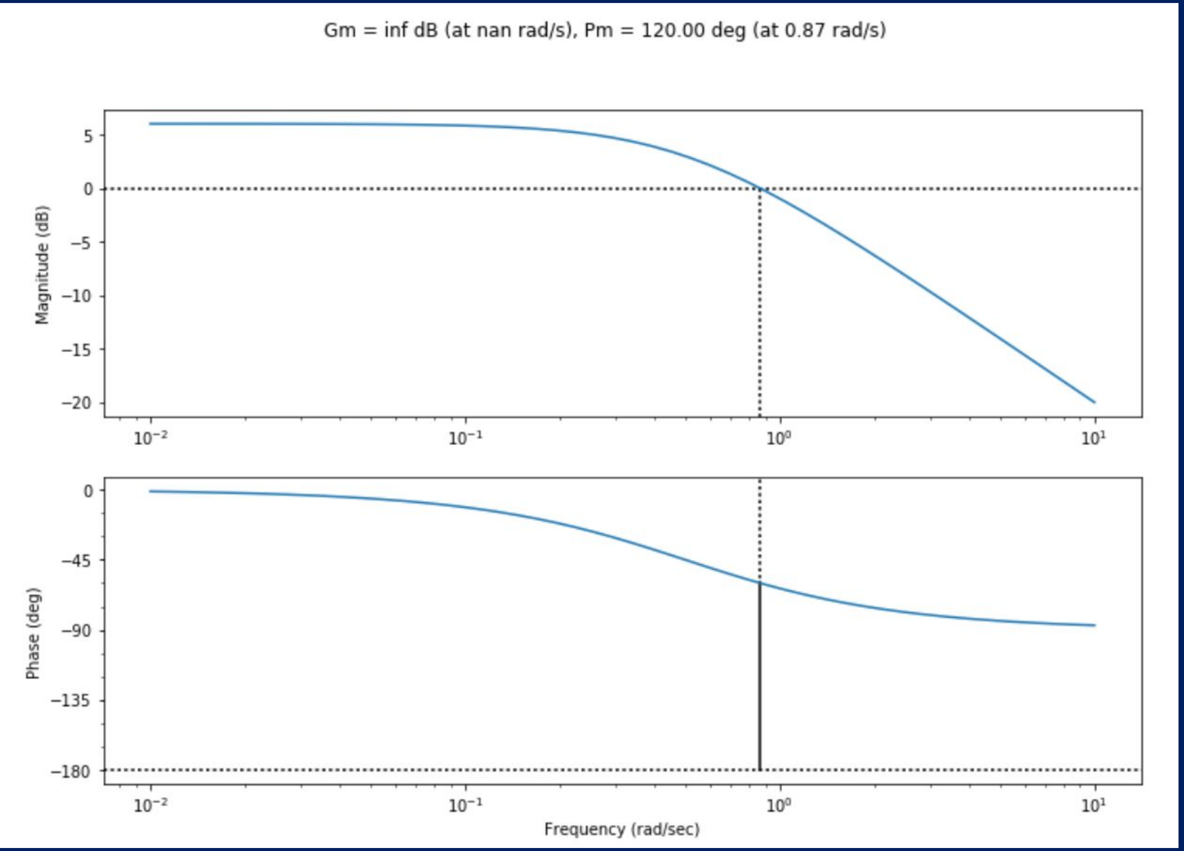
Delays therefore causes that we need to lower the bandwidth or speed of controller.
If we do not lower the bandwidth so we can loose the stability of the system.

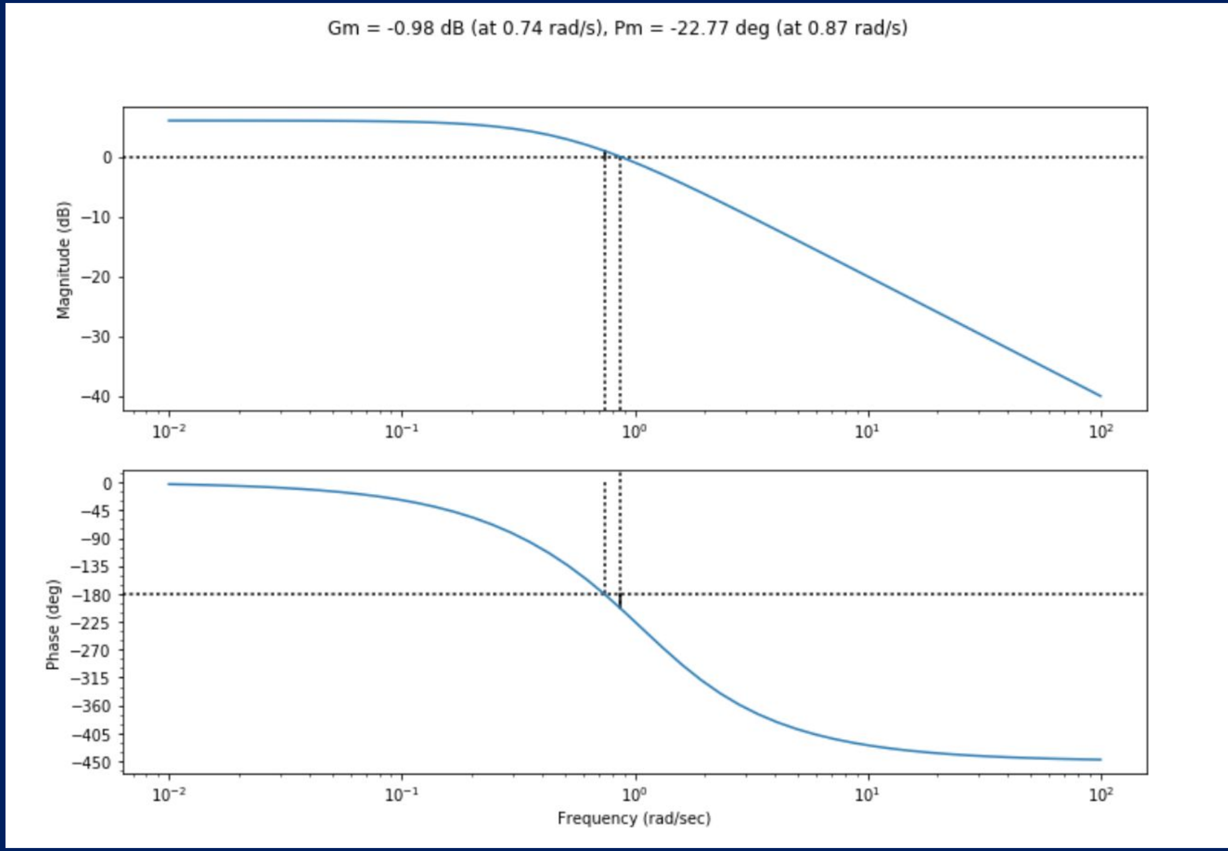
Perfect: system without delay
Gm – Infinity
Pm – 120deg OK

$$\frac{2}{2s+1}$$

SYSTEM UNSTABLE TOO MUCH DELAY
Gm – 0.98
Pm – minus 22.77 deg. BAD

$$\frac{2}{2s+1}e^{-3t}$$





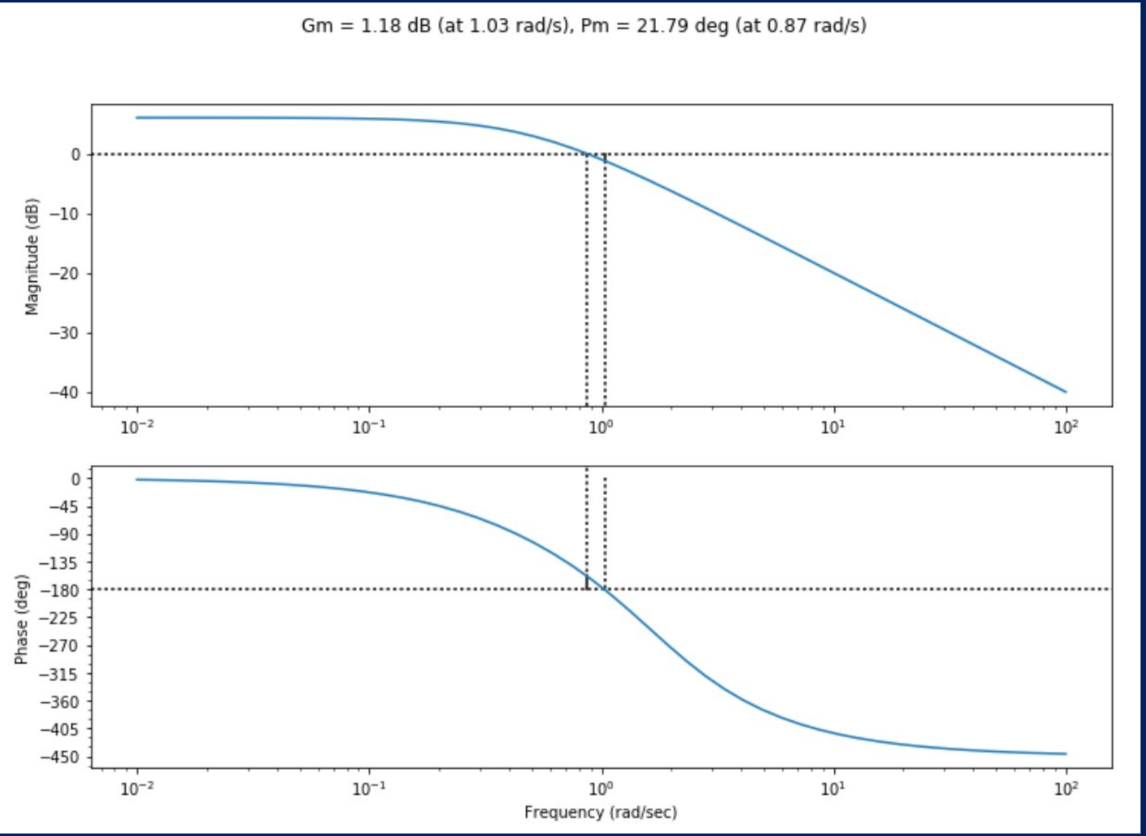
In order to increase the stability we need to shift the bandwidth of the system (lower cross-over frequency) so we need to decrease the gain so the system will slower and less responsive !!!

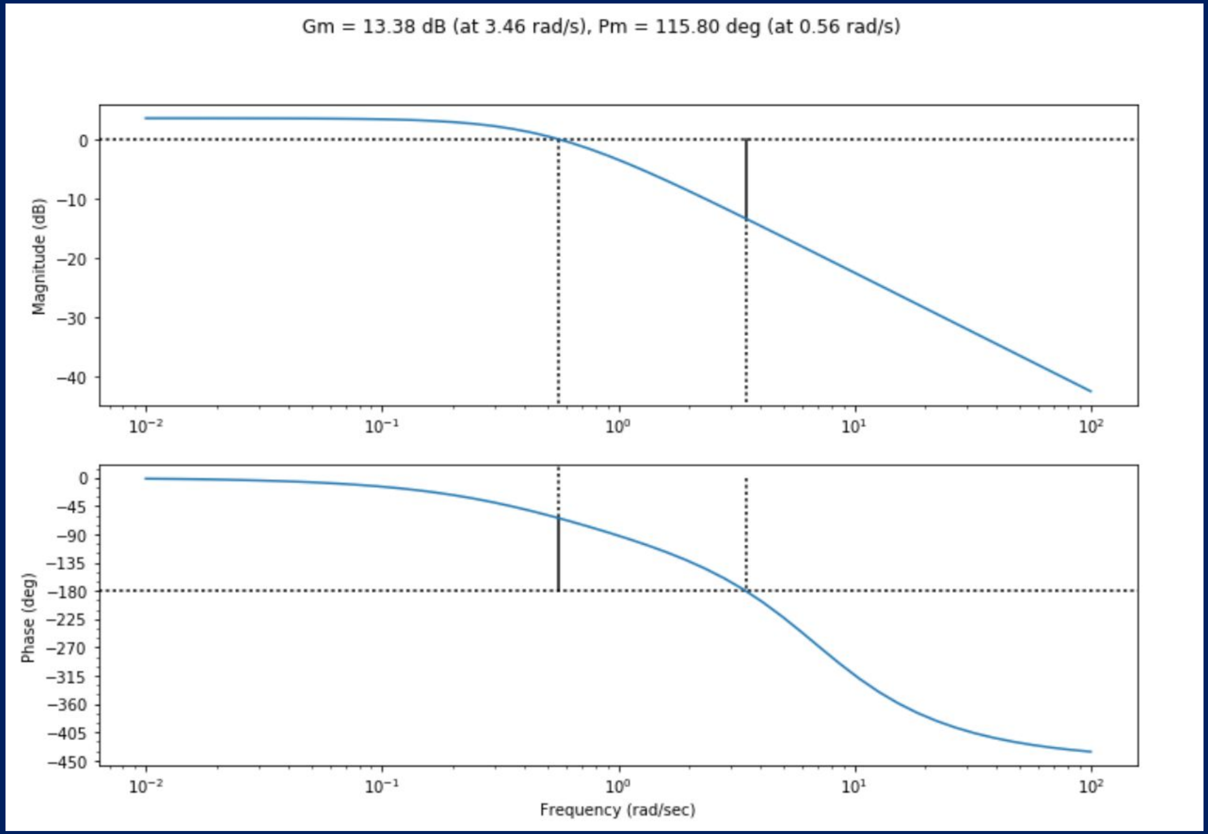
System with delay: 0.5s
Gm drops to 1.16dB
Pm drops to 21.8deg

$$\frac{2}{2s+1}e^{-0.5t}$$

System with delay: 0.5s

$$\frac{1.5}{2s+1}e^{-0.5t}$$



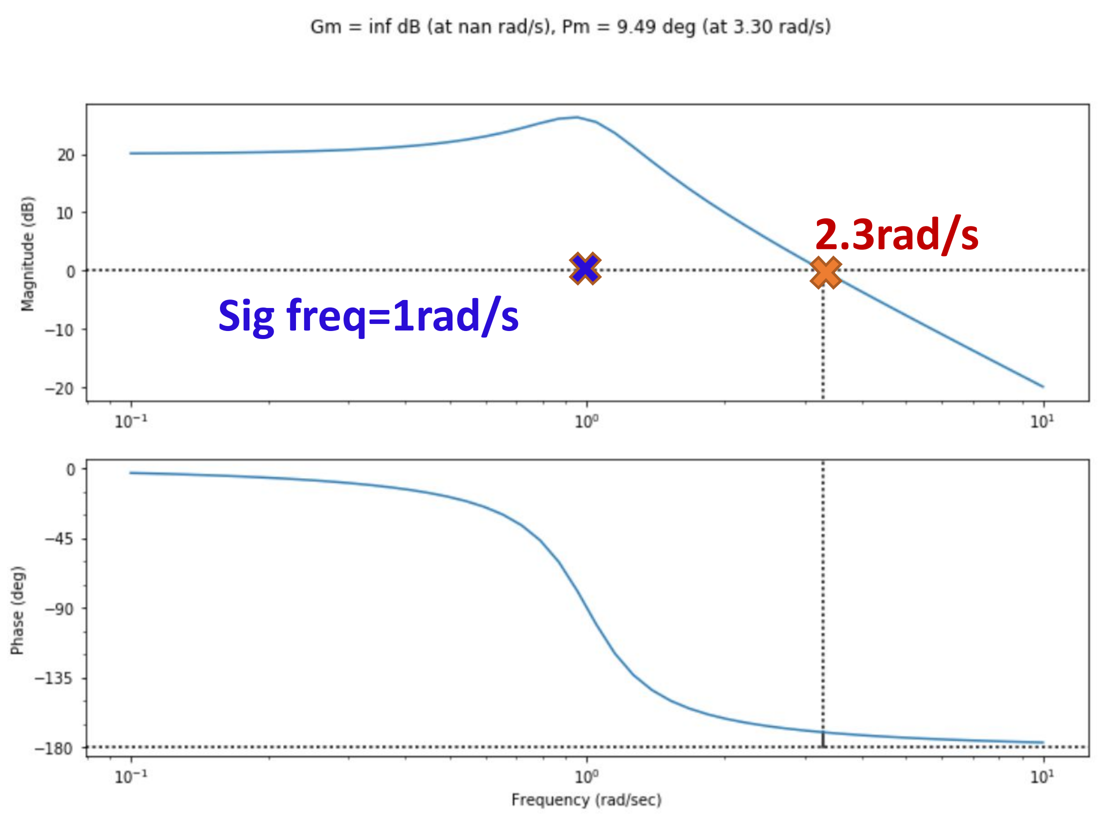


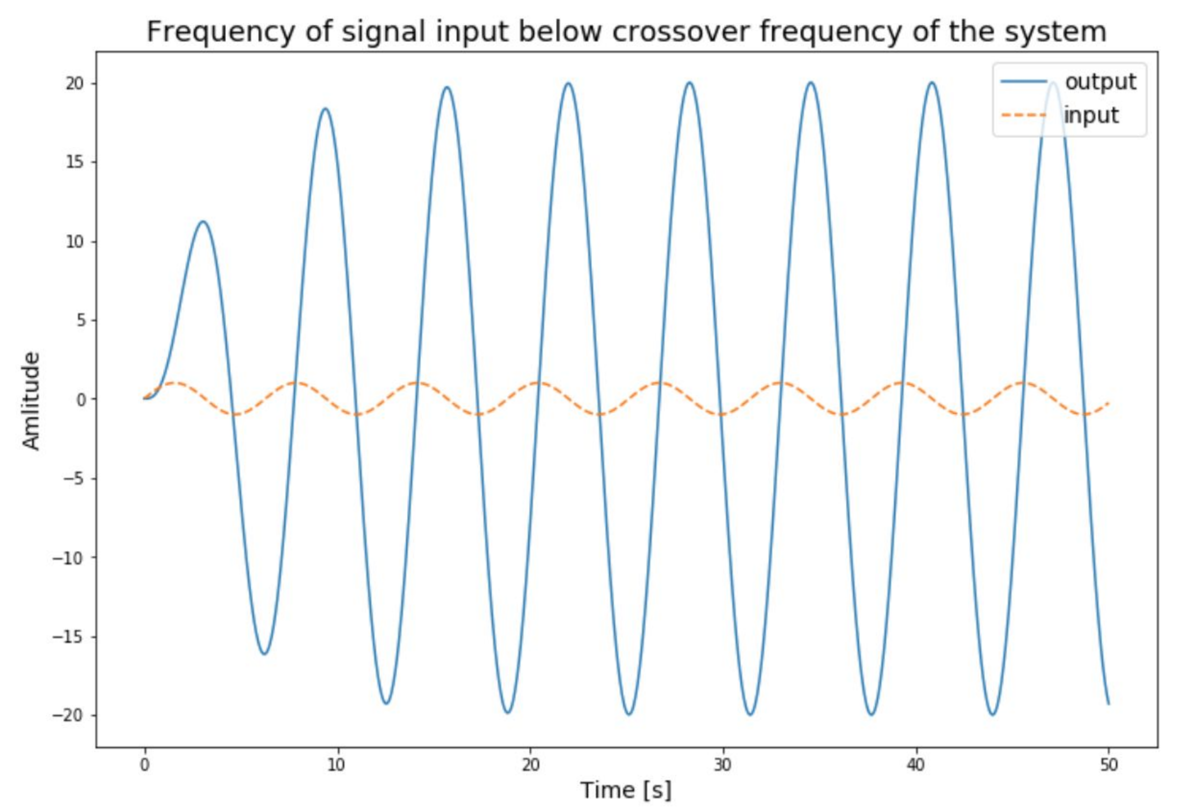
Bandwidth system limitation

Transfer system function $\dfrac{10}{s^2 + 0.5s + 1}$

sig_input(t) = sin(1*t) => 1rad/s
COW frequency = 2.3 rad/s
PM is too low = 9.49deg because we introduced system delay

The frequency of the input signal is below crossover (natural frequency = 2.3rad/s) of the system so amplitude for signal out follows the Body plot magnitude and in this case will be amplified by factor 20. Some time delay (phase shift) can be noticed = > see Body plot for Phase change

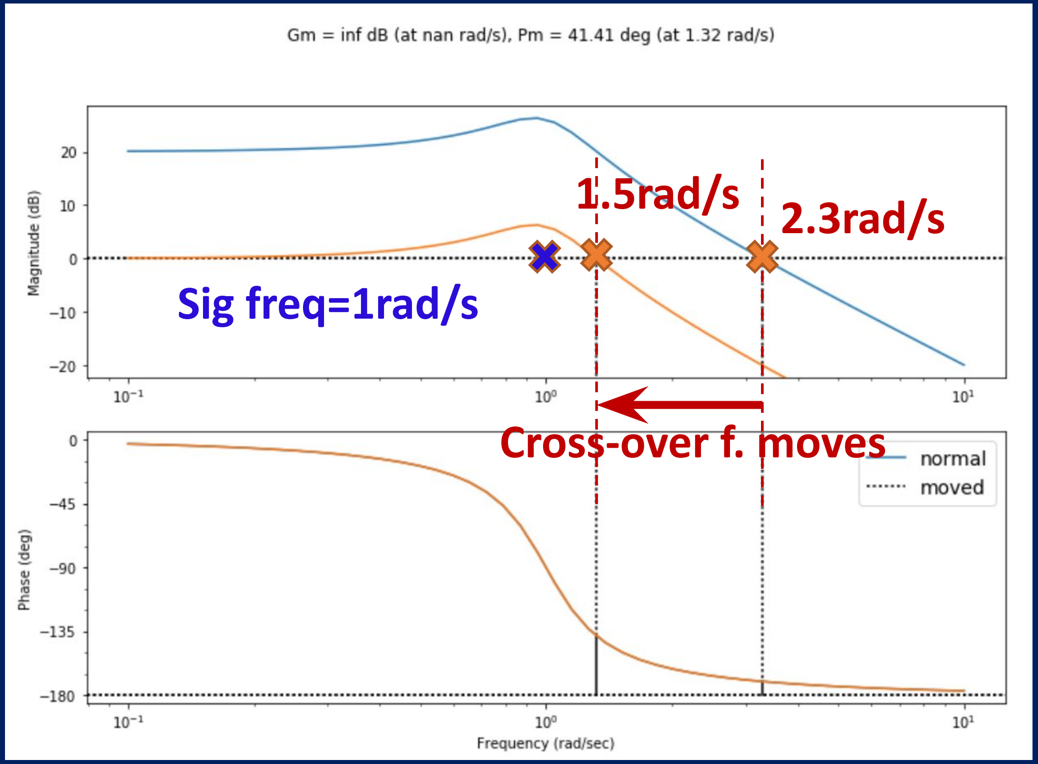


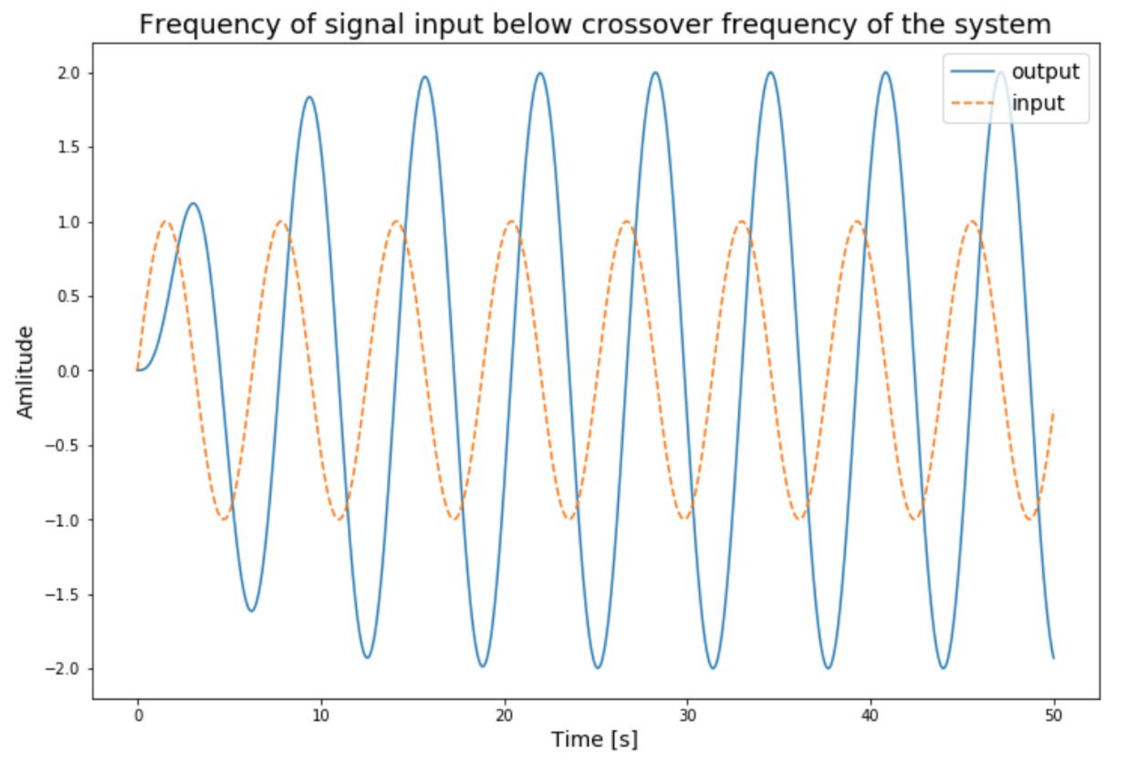


Now (because phase compensation due to delay in the system) we have to move crossover frequency to the left so our input signal will be amplified only by factor 2 and damped for these frequencies which are higher then new cross-over (moved to left frequency) 1.5rad/s.

sig_input(t) = sin(1*t) => 1rad/s
COW frequency = 1.5 rad/s
PM = 40.41deg

The system is now less responsive and slower. Higher frequencies which "build" the stepper signal (which grows faster) are removed !!!





NOTCH FILTER (narrow band filter)

We need sometimes to remove only very specific noise (signal frequency) from power supply 50Hz or from flexible equipment (gears, joints, coupling, etc) in order to feed controller only with process (useful) signal.

Low pass
cut-off=1rad/s

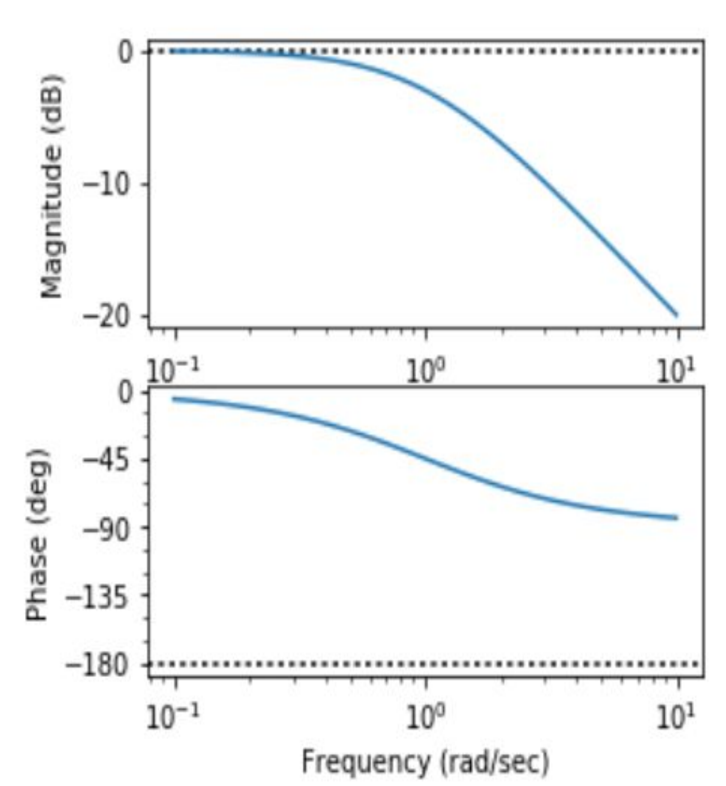
$$\frac{1}{s+1}$$

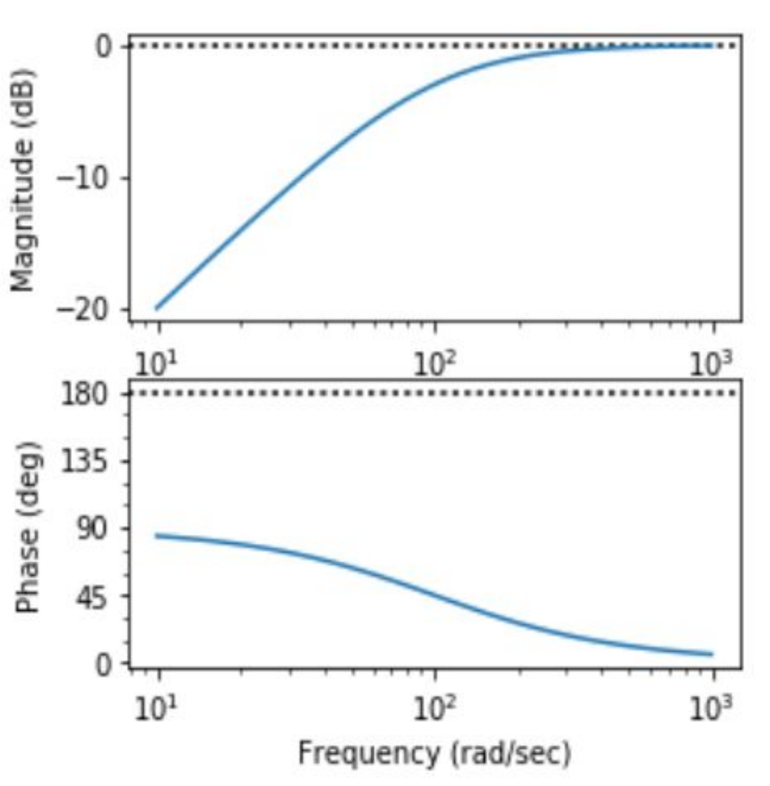
High pass
cut-off=100rad/s

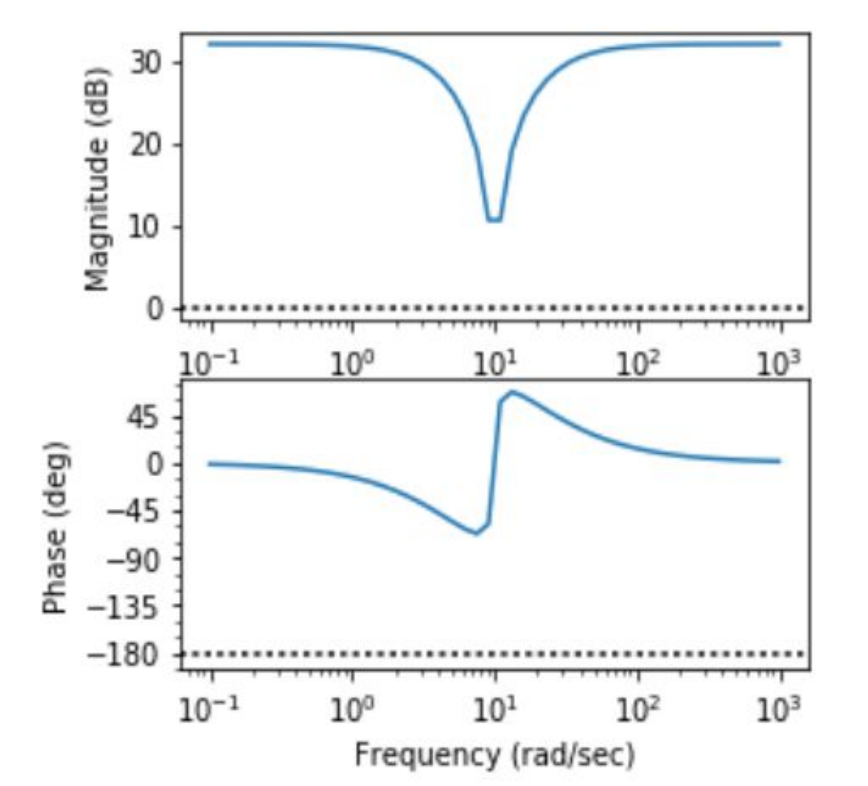
$$\frac{s}{s+100}$$

Notch filter
Pass band = 5 – 20 rad/s

$$\frac{400s^2+400s+40000}{10s^2+250s+1000}$$



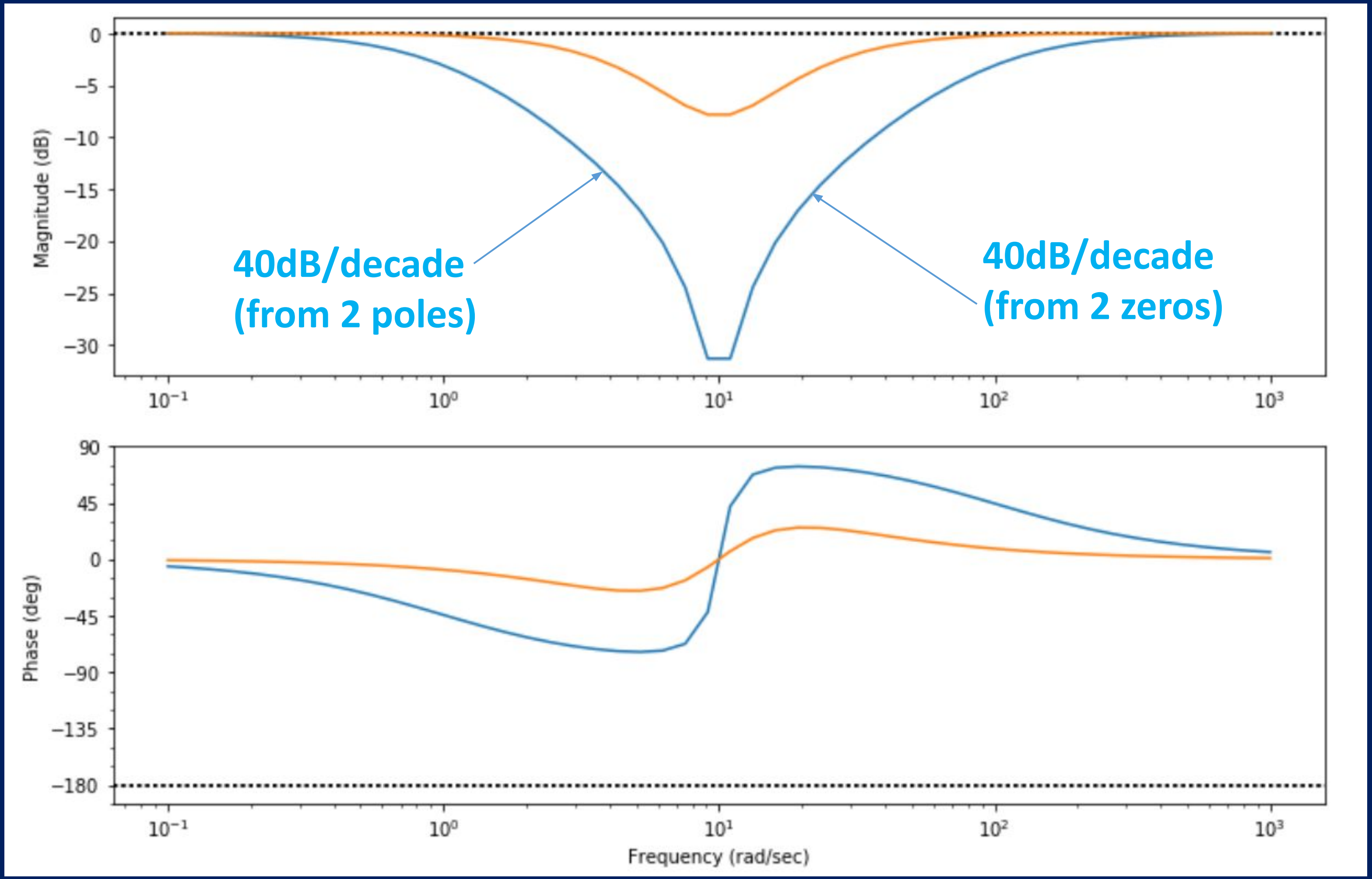




How to build a Notch filter?

The idea is to connect the low and high filter. But how to control depth of filter, while narrowing the pass band (keeping the depth in order to filter out requested frequency)?

Low and high pass filters. **Blue** original. **Orange** cut-off frequency for low pass shifted from 1rad/s to 5rad/s



WE USE STANDARD LOW PASS FILTER SECOND ORDER equation.

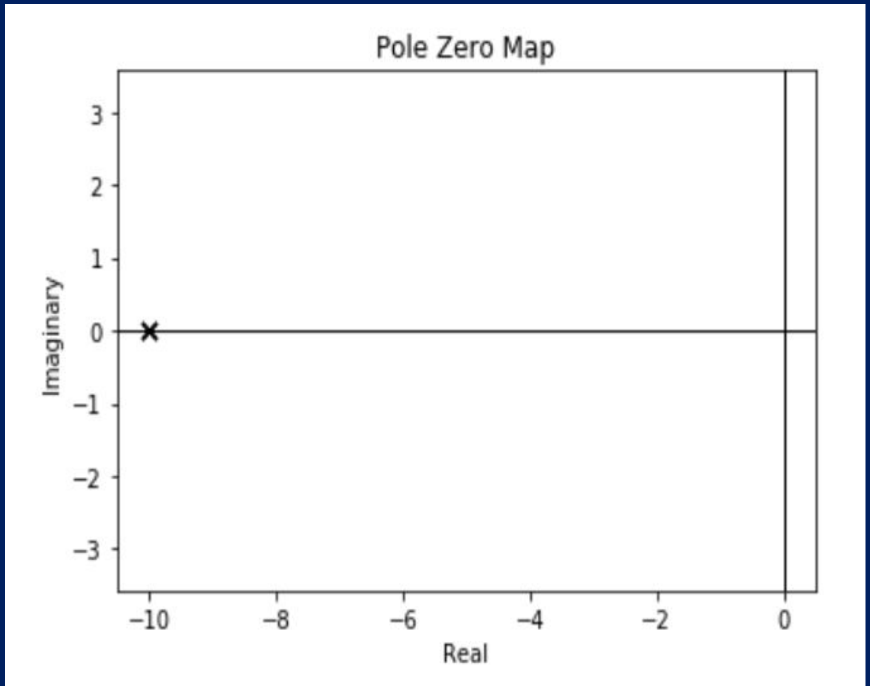
WE TRY TO IMPROVVE DEPTH (INCREASE THE SLOPE OF GAIN FROM 40dB/decade to MAX

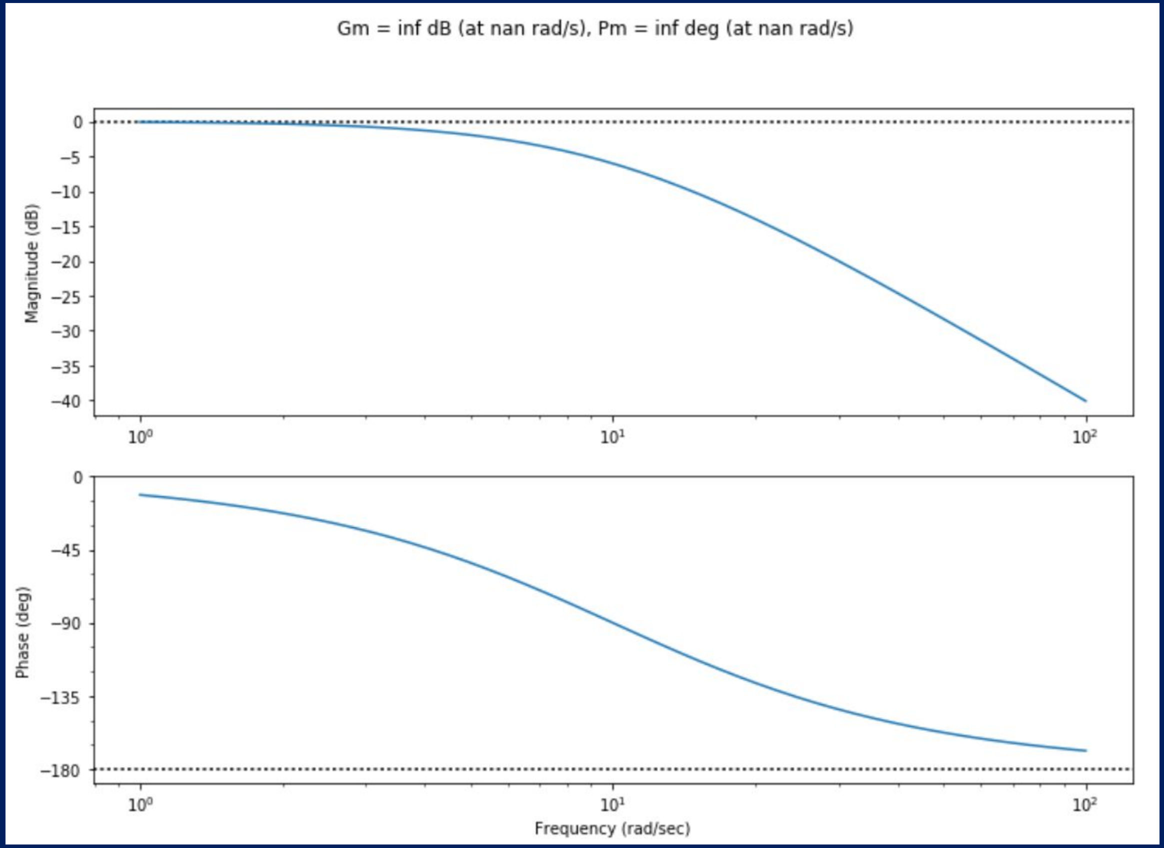
$$\frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2}$$

$\omega_n = 10 \ - natural\ frequency$

$\zeta = 1 \ - dampning$

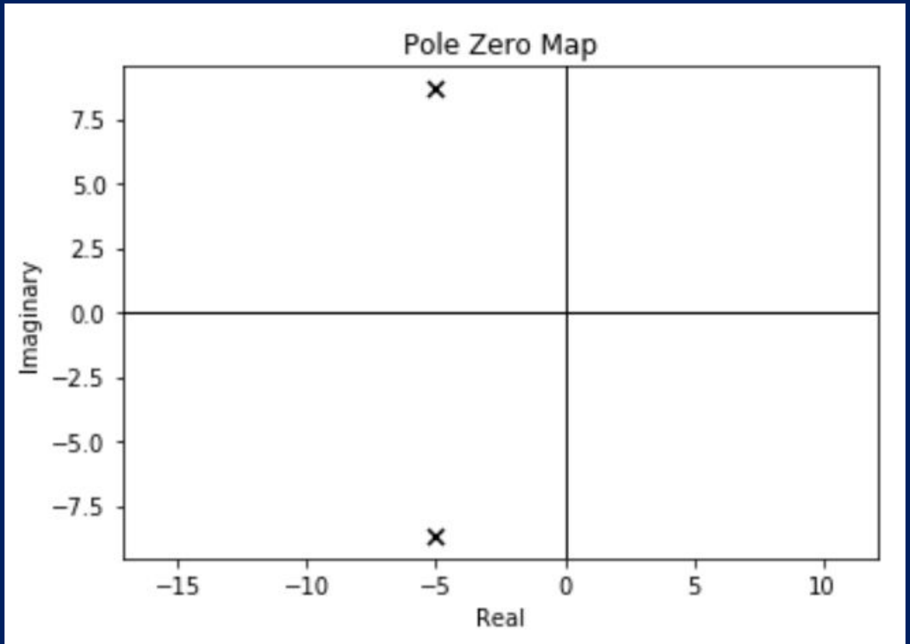
$$\frac{100}{s^2 + 20s + 100}$$

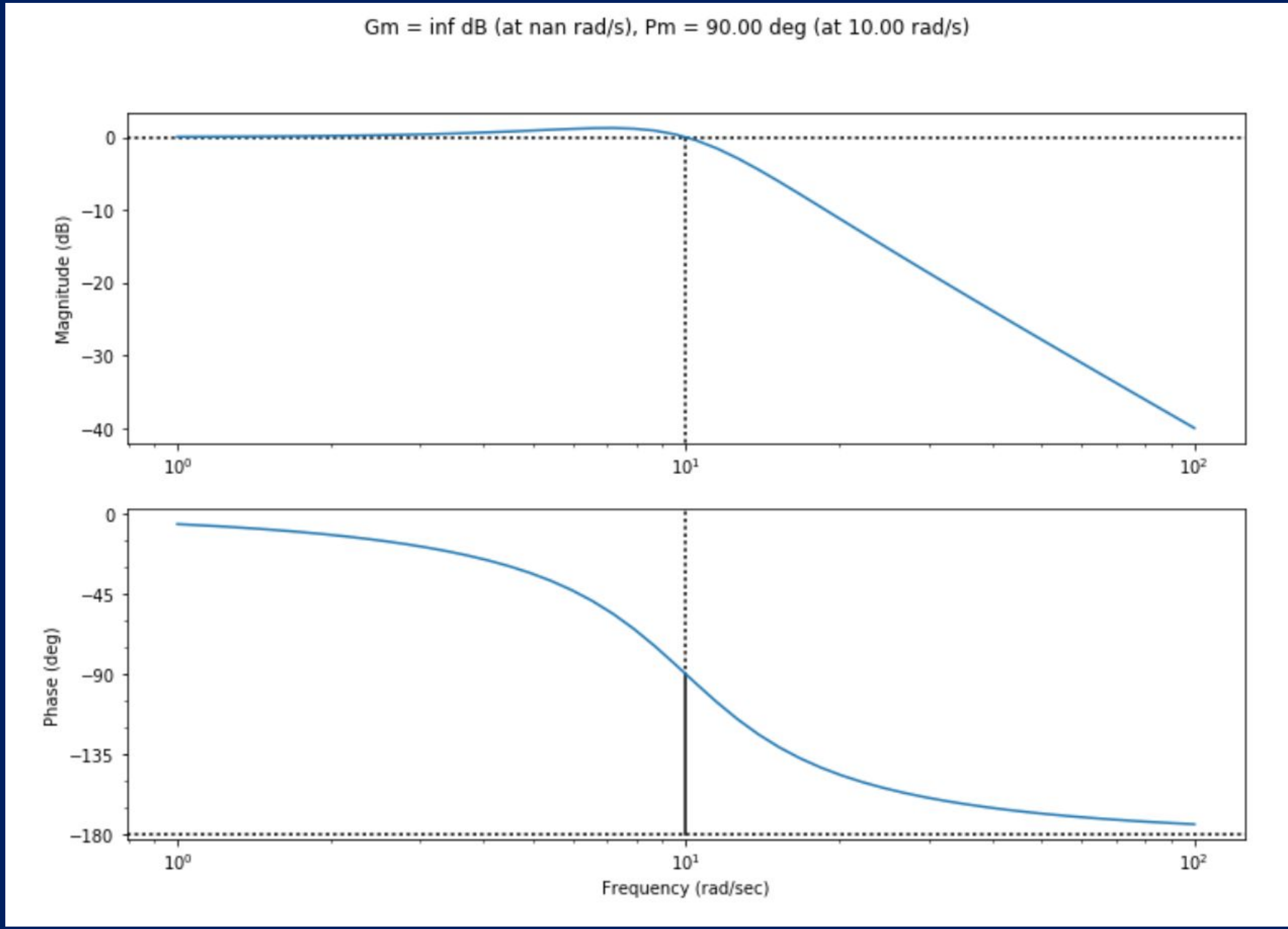




We lower damping to 0.5

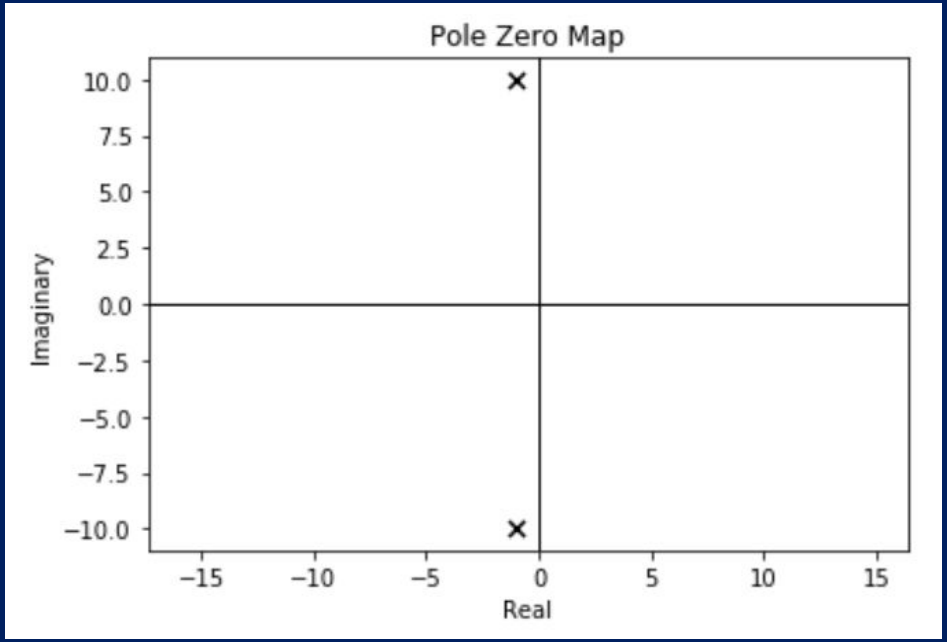
$$\frac{100}{s^2 + 10s + 100}$$

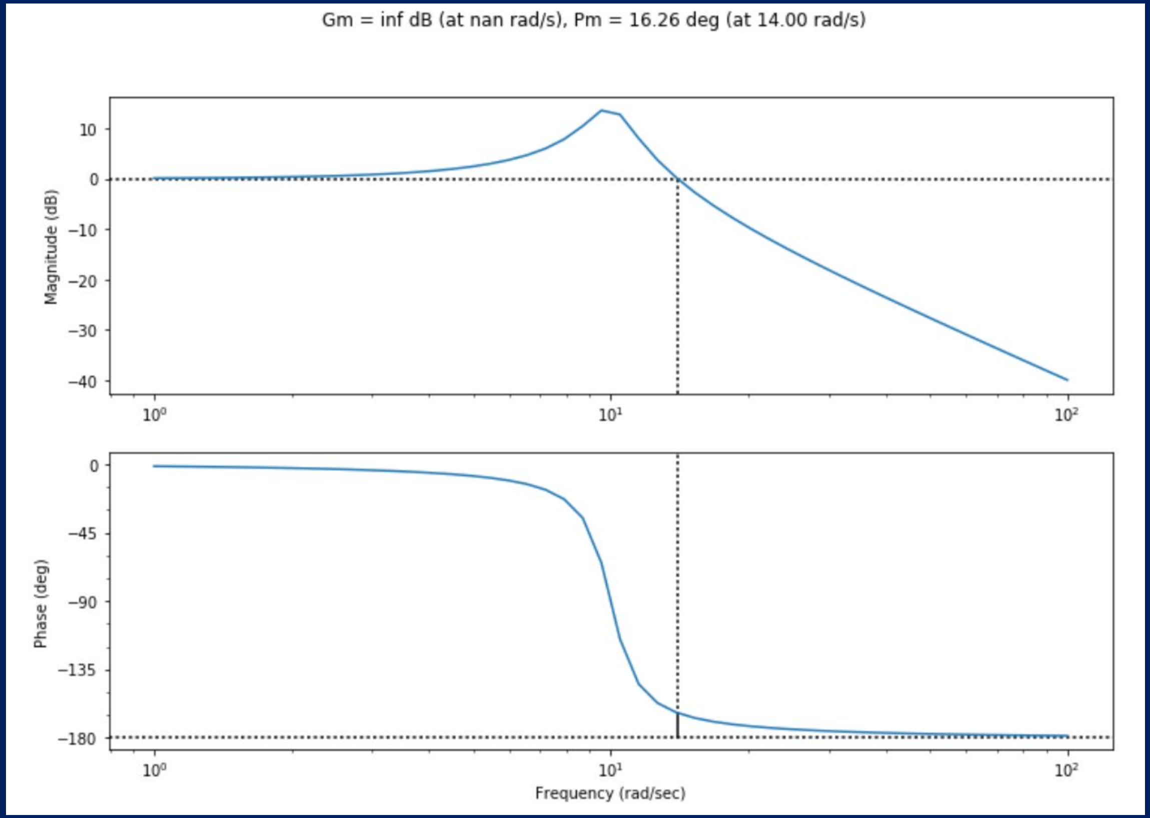




We lower damping to 0.1

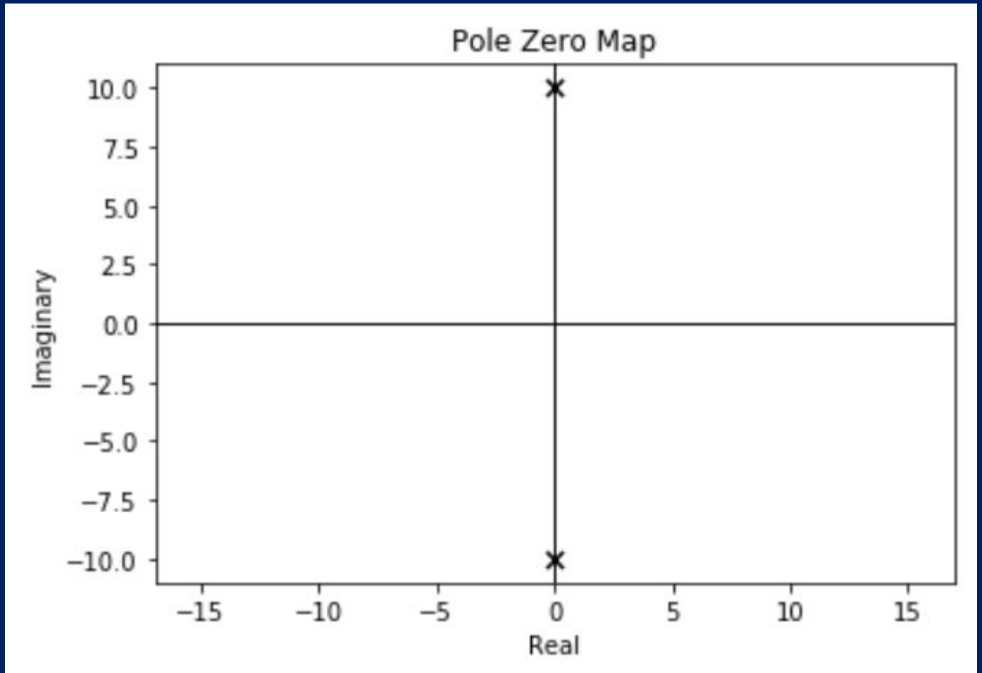
$$\frac{100}{s^2 + 2s + 100}$$

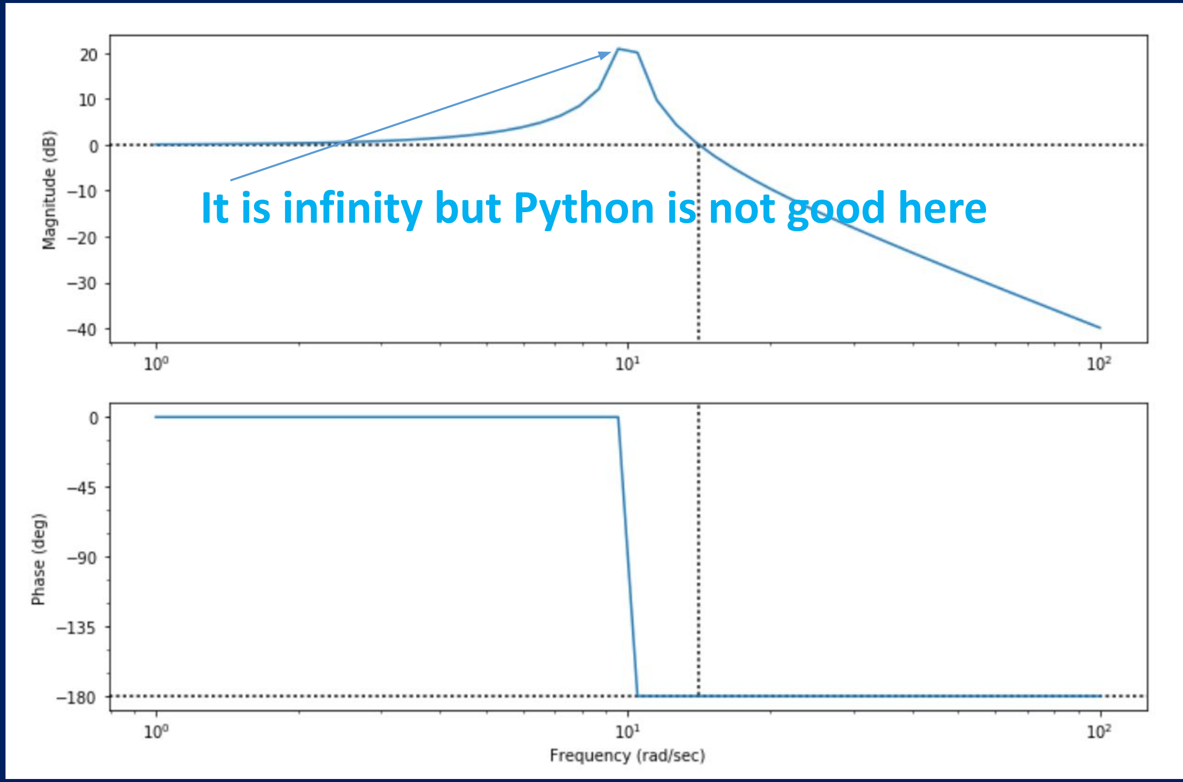




We lower damping to 0

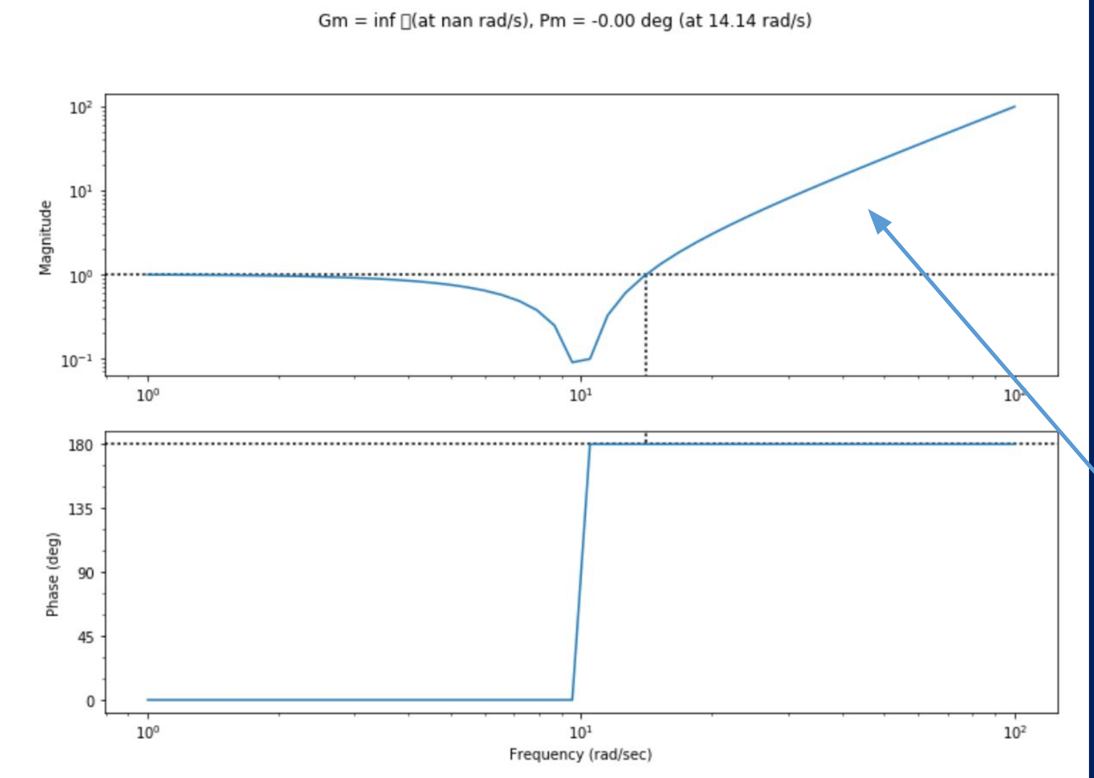
$$\frac{100}{s^2 + 100}$$





We flips this guy…

It is a Notch filter but flipped. Extremely high gain in natural frequency which normally we would like to avoid.



It is almost perfect with this flipped tranfer function but we have two problems:

1. high-frequency signals which are amplified by 40dB/decade since we have only two zeros (unanswered) and
2. the transfer function can not be physically realized because the numerator is greater then denominator

WE FIX THESE TWO PROBLEMS BY ADDIND TWO POLES.
Each pole will drag the high frequency magnitude down by 20dB/decade and flat our frequency response over natural frequency (crossover).

HOW to choose poles?

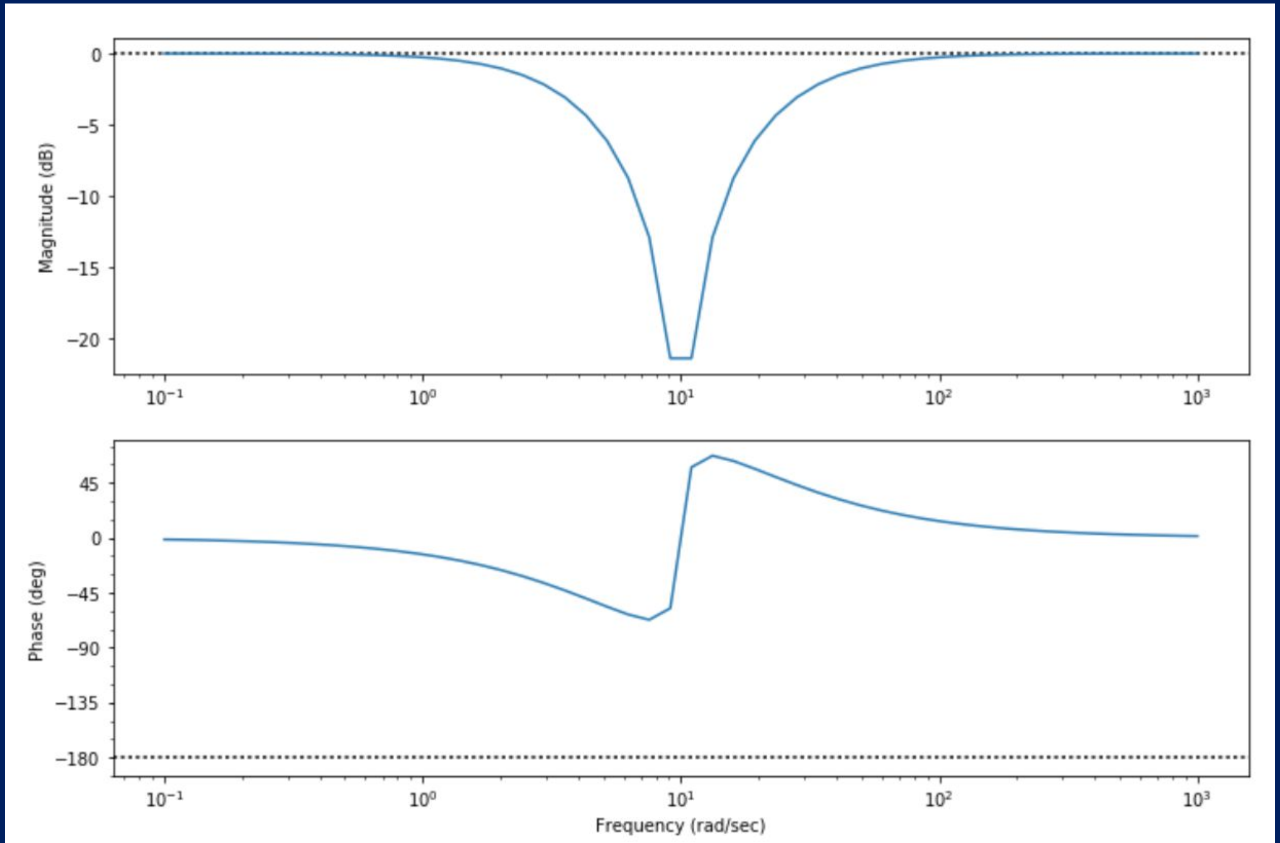
we use formula for pole which should be located below and above natural frequency.

Pole above cross over frequency $= \dfrac{a \cdot \omega_n}{s + a \cdot \omega_n}$

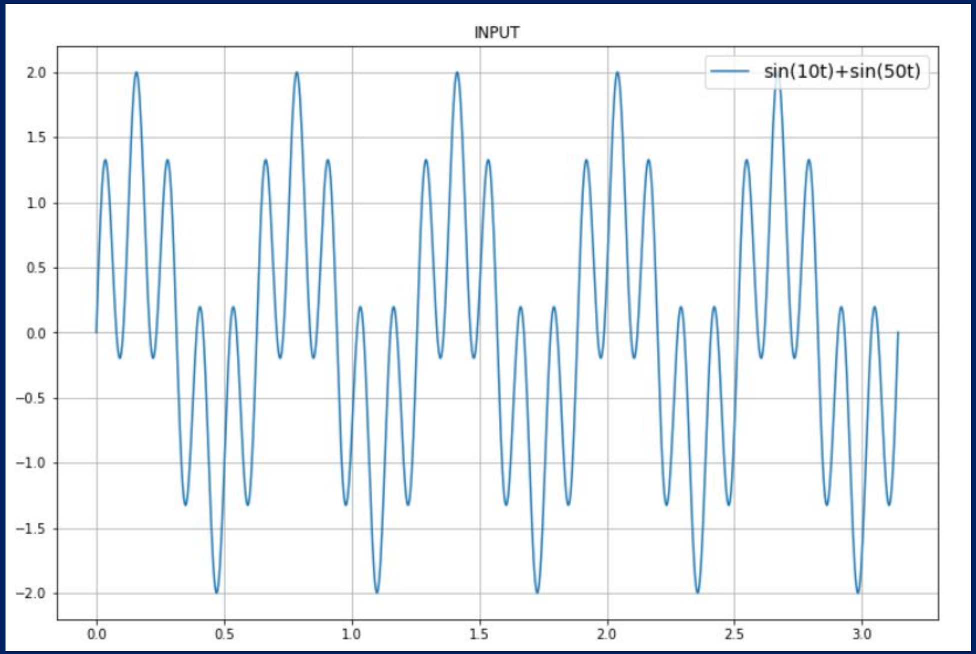
Pole lower cross over frequency $= \dfrac{\frac{\omega_n}{\mathrm{a}}}{s + \frac{\omega_n}{\mathrm{a}}}$

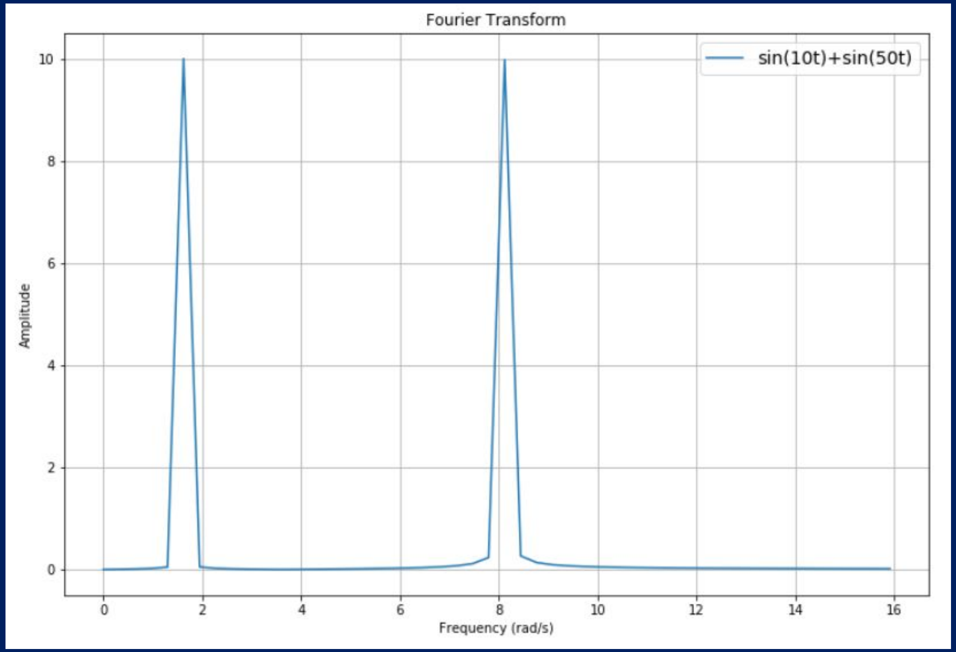
a = 2 => factor pole a time lower and higher than natural frequency

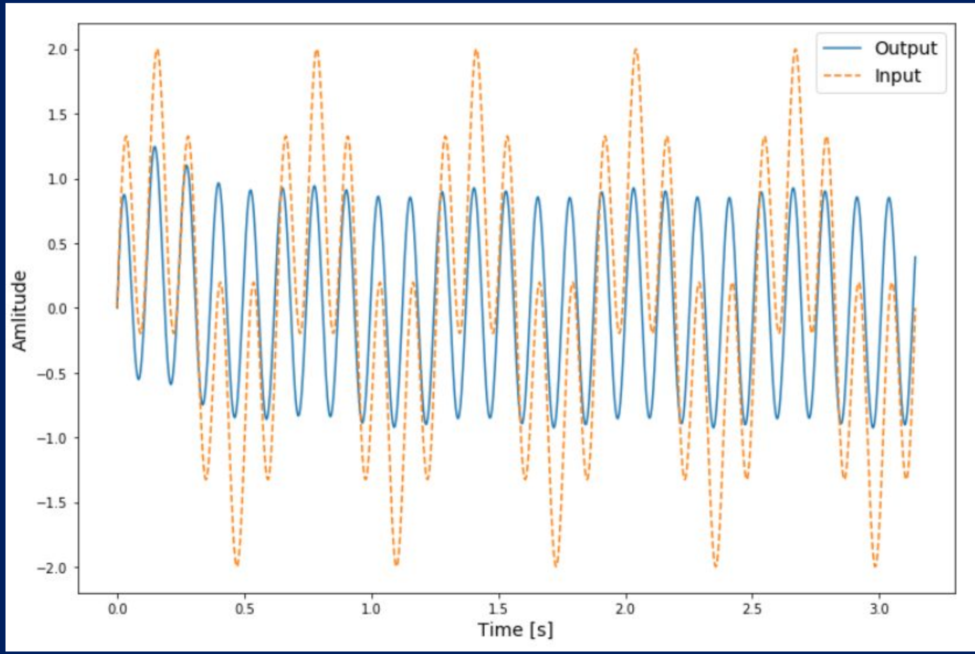
Notch filter

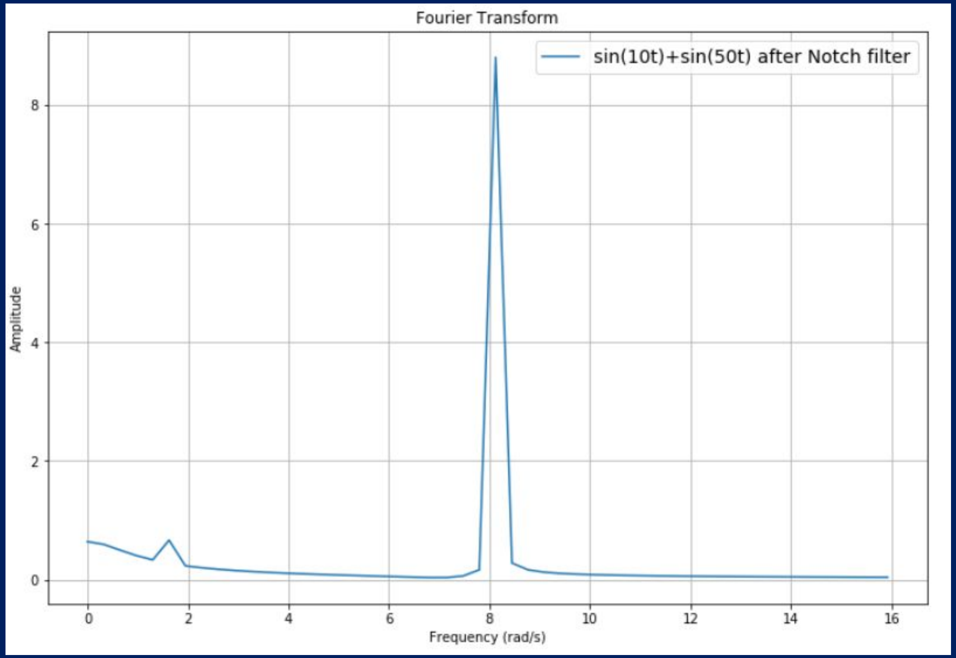


Notch filter in action. Input signal sin(10t) + sin(50). Filter cuts input signal frequency = 10rad/s as designed










I'LL BE BACK